

NOTES

POWER FUNCTIONS OF THE SIGN TEST AND POWER EFFICIENCY FOR NORMAL ALTERNATIVES¹

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1. **Summary.** Power functions are tabulated for the sign test for various sample sizes and α near .05 and .01. Several of these power functions are compared with the power function of the t -test for samples from normal populations by means of a power efficiency function. The results indicate decreasing power efficiency for increasing sample size, for increasing level of significance and for increasing alternative.

2. **Power function.** The power of the two-sided sign test for level of significance, α , is given by:

$$(1) \quad \lambda(p) = \sum_{j=0}^i \binom{N}{j} [p^j(1-p)^{N-j} + p^{N-j}(1-p)^j]$$

where i is the largest integer such that

$$(2) \quad \sum_{j=0}^i \binom{N}{j} (1/2)^N \leq (1/2)\alpha$$

and N is considered fixed [5]. Here, p is the alternative population proportion. Values for $\lambda(p)$ may be obtained readily from a table of the cumulative binomial [1] or tables of the incomplete beta function [2] since

$$\sum_{j=0}^i \binom{N}{j} x^j(1-x)^{N-j} = I_{1-x}(N-i, i+1).$$

Beyond the range of these tables the approximation of Camp [3] can be used with great accuracy. The maximum i which satisfies (2) is tabulated as r in Table I of reference [5] for $\alpha = .01$ and $\alpha = .05$. Tables I and II of this paper give the power for these critical values. Since $p = .50$ is the null hypothesis, the values in the column headed $p = .50$ in Tables I and II of this paper give the actual level of significance ($\leq .01$ or $\leq .05$) of each test. At the foot of the tables are the normal alternatives corresponding to the alternative p , that is, δ is defined by the relation $1 - F(\delta) = p$ where $F(x)$ is the cumulative zero mean unit variance normal distribution. For normal alternatives Tables I and II may be entered either with p or δ . For nonnormal alternatives the tables must, of course, be entered with p .

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TABLE I
Power for Sign Test ($\alpha \leq .05$)

ρ50	.45	.40	.35	.30	.25	.20	.15	.10	.05
N			.55	.60	.65	.70	.75	.80	.85	.90	.95
N	r										
(5)	(0)	.06250	.06878	.08800	.12128	.17050	.23828	.32800	.44378	.59050	.77378
6	0	.03125	.03598	.05075	.07726	.11838	.17822	.26221	.37716	.53144	.73509
7	0	.01562	.01896	.02963	.04967	.08257	.13354	.20973	.32058	.47330	.69834
8	0	.00781	.01005	.01745	.03209	.05771	.10013	.16777	.27249	.43047	.66342
9	1	.03906	.04760	.07435	.12248	.19644	.30045	.43623	.59948	.77484	.92879
10	1	.02148	.02776	.04804	.08649	.14945	.24405	.37581	.54430	.73610	.91386
(10)	(2)	.10938	.12895	.17958	.26643	.38437	.52601	.67788	.82021	.92981	.98850
11	1	.01172	.01614	.03097	.06079	.11304	.19711	.32212	.49219	.69736	.89811
12	2	.03857	.05002	.08625	.15214	.25302	.39071	.55835	.73582	.88913	.98043
13	2	.02246	.03105	.05922	.11354	.20255	.33261	.50165	.69196	.86612	.97549
14	2	.01294	.01916	.04040	.08407	.16086	.28113	.44805	.64791	.84164	.96995
15	3	.03516	.04875	.09243	.17318	.29696	.46130	.64816	.82266	.94444	.99453
16	3	.02127	.03158	.06609	.13406	.24539	.40499	.59813	.78939	.93159	.99300
17	4	.04904	.06820	.12852	.23542	.38879	.57390	.75822	.90129	.97786	.99884
18	4	.03088	.04598	.09545	.18888	.33269	.51867	.71635	.87944	.97181	.99845
19	4	.01921	.03074	.07025	.15011	.28224	.46543	.67329	.85556	.96481	.99799
(20)	(4)	.01182	.02039	.05127	.11825	.23751	.41484	.62965	.82985	.95683	.99743
20	5	.04139	.06177	.12721	.24571	.41641	.61718	.80421	.93269	.98875	.99967
(20)	(6)	.11532	.15135	.25648	.41815	.60827	.78581	.91331	.97806	.99761	.99997
25	7	.04329	.06968	.15476	.30626	.51187	.72651	.89088	.97453	.99774	.99998
30	9	.04277	.07442	.17714	.35764	.58882	.80341	.93891	.99034	.99955	1.00000-
35	11	.04096	.07712	.19577	.40198	.65156	.85789	.96564	.99633	.99991	1.00000-
40	13	.03848	.07848	.21156	.44079	.70325	.89776	.98059	.99860	.99998	1.00000-
45	15	.03570	.07894	.22517	.47519	.74622	.92470	.98900	.99946	1.00000-	
50	17	.03284	.07877	.23706	.50598	.78219	.94488	.99374	.99980	1.00000-	
60	20	.02734	.07722	.25839	.55903	.83818	.97020	.99796	.99997	1.00000-	
70	26	.04139	.11635	.36009	.69503	.92220	.99163	.99974	1.00000-		
80	30	.03290	.10395	.36877	.72353	.94125	.99542	.99991	1.00000-		
90	35	.04460	.14612	.46008	.81223	.97256	.99878	1.00000-			
100	39	.03520	.13519	.46206	.82758	.97900	.99932	1.00000-			
Normal alter-											
natives											
δ	0	.1257	.2534	.3853	.5244	.6745	.8416	1.0364	1.2816	1.6449	
$\sqrt{2}\delta$	0	.1777	.3583	.5449	.7416	.9539	1.1902	1.4657	1.8124	2.3262	

3. Power efficiency. Discussion of the power of the sign test for normal alternatives was given in [4] for large N . This paper obtains $100(2/\pi) = 63.7$ per cent as the efficiency. Reference [5], by a rough coincidence of the power function of the sign test for a sample of N observations with the power function of the t -test

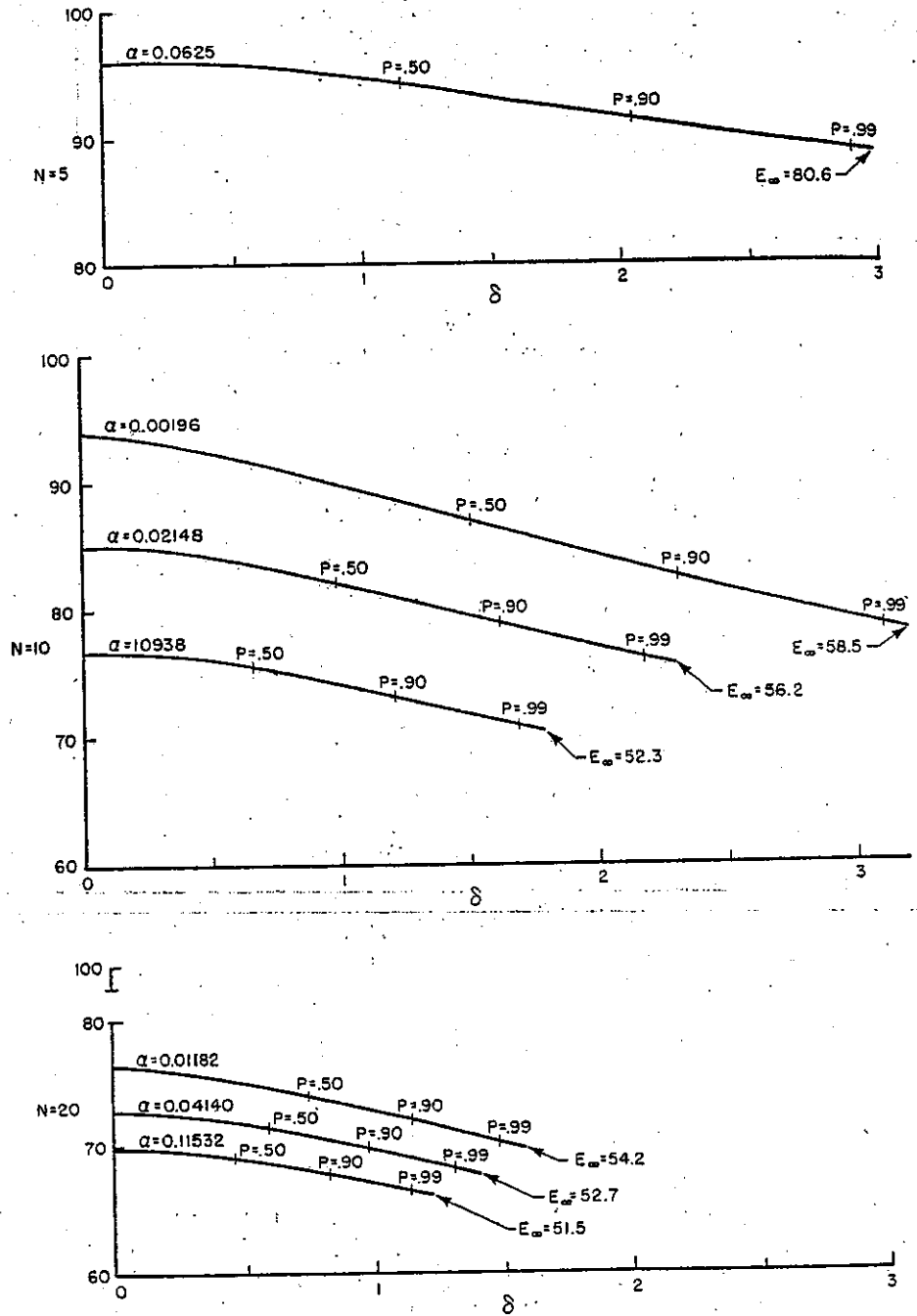


FIG. 1

will be used to refer to any temperature change that produces a significant disturbance in the normal functions of a freshwater teleost and thus decreases the probability of survival. This definition is similar to that proposed by Brett (1958) and assumes that the response is a stochastic variable that can be measured quantitatively. It also assumes that thermal stress is harmful. Although this is a reasonable assumption for the individual fish it is not always true for the population or community. For example, increased mortality in a crowded population may be beneficial to the fish population as a whole if space and/or food resources are limited. An extreme example is the death of the entire population through the severe stress of a thermal discharge, but even this may be beneficial to other members of the community.

It has been traditionally assumed in ecology that increasing complexity or diversity in a community produces increasing stability (e.g. Elton, 1958), but a more recent proposition is that complexity begets instability not stability, and that ecological communities persist despite, not because of, their complexity (e.g. May, 1973, 1976; Pimm and Lawton, 1980). It is, therefore, not surprising that stress responses at the population and community level are very complex and that little is known about the response of a freshwater ecosystem (for general reviews of stress and ecosystems see Odum, 1967, 1974; Slobodkin and Sanders, 1969; Gibbons, 1976; Lugo, 1978; Leffler, 1978). Therefore the present paper considers thermal stress only at the level of the individual fish.

The responses of a fish to stress can be broadly classed as either primary or secondary. Primary responses include neuro-endocrine and endocrine reactions which are reviewed in detail elsewhere in this volume (see contributions by Donaldson, 1981; Mazeaud and Mazeaud, 1981). Examples of primary responses to thermal stress include work on juvenile coho salmon, *Oncorhynchus kisutch* (Wedemeyer, 1973), goldfish, *Carassius auratus* (Fryer, 1975), juvenile sockeye salmon, *Oncorhynchus nerka* (Mazeaud *et al.*, 1977), and juvenile cutthroat trout, *Salmo clarki* (Strange *et al.*, 1977). As the fish were subjected to rapid increases of 10–15°C in these experiments, it is not surprising that the primary responses were so large. Such temperature increases rarely occur naturally in fresh water, and it would be of more ecological value to know the primary responses to smaller temperature changes at different acclimation temperatures and the level of temperature change at which there is no significant primary response.

There are numerous secondary responses to thermal stress and these include disturbances in osmotic and ionic regulation, metabolic processes, growth, reproduction and behaviour. The ultimate response is death. Metabolic and osmoregulatory disturbances during stress are effected by neuro-humoral changes (see reviews by Love, 1970; Maetz, 1974; Fontaine,