TECHNICAL MEMORANDUM: SHASTA RIVER ALGAE BOX MODEL

TO: Matt St John, North Coast Regional Water Quality Control Board

FROM: Mike Deas, Watercourse Engineering, Inc.

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Michael Johnson, University of California, Davis

RE: Shasta River Periphyton Analysis

DATE: August 16, 2005

Introduction

While modeling the Shasta River, it was determined that exploring the connection between nutrient levels in the river and potential primary production might lead to more accurately modeled dissolved oxygen. Therefore, an existing model used to predict phytoplankton biomass was altered and employed to determine the periphyton biomass in Shasta River based on limiting factors such as light and nutrients, as well as on respiration and mortality rates. Scouring and shading were also included. Such models are simplifications of natural systems, nonetheless, can provide insight into potential system dynamics. Given the limited available information on the Shasta River, the model is applied herein as a screening tool to determine potential cause and effect relationships for variable water quality conditions.

Model Approach

Existing Model

The existing mass balance model was a volume-based model that calculated the concentration of algae in the water of the reach, called phytoplankton. Equation (1) represents the original differential equation representing the algal growth over time.

$$V\frac{dP}{dt} = V(\mu - R_P - D_P)P - A\nu_s P + Q_{in}P_{in} - Q_{out}P \qquad (1)$$

Where:

V = volume (m^3)

P = phytoplankton biomass (μg/l)

μ = algal growth rate (1/d)

R_P = algal respiration rate (1/d)

D_P = algal predatory and non-predatory mortality (1/d)

A = bed area (m^2)

v_s = algal settling rate (m/d)

Q_{in} = inflow rate (m^3/d)

P_{in} = inflow algal concentration ((µg/l)

Q_{out} = outflow rate (m^3/d)

A forward difference approximation was employed to use the equation in an iterative form, creating Equation (2), presented below. $P_{t+\Delta t}$ represents the phytoplankton concentration at the future time, P_t represents the phytoplankton concentration at the current time, and Δt is the time interval; thus a simple marching scheme can be implemented to solve for $P_{t+\Delta t}$.

$$P_{t+\Delta t} = P_t + \left(\frac{\Delta t}{V}\right)\left(V(\mu - R_P - D_P)P_t - A v_s P_t + Q_{in} P_{in} - Q_{out} P_t\right) \quad (2)$$

Where:

Δt = change in time (d)

Shasta River Benthic Algae Model

To modify the existing algae model to a benthic algae model, several changes were made. The state variable was changed from phytoplankton, measured in volumetric concentration to benthic algae, measured in biomass per area. Limiting factors were calculated and, along with the maximum growth rate, used to create an apparent growth rate. A grazing coefficient was added along with the respiration and mortality coefficients. The settling component of the equation, $A v_s P_t$, was removed, as benthic algae cannot settle. The inflow algae concentration component was removed. Altering the outflow algae concentration component created a scouring term. The final mass balance equation for iteration of the Shasta River Benthic Algae Model is presented below (Equation (3)).

$$P_{t+\Delta t} = P_t + \Delta t\left((\mu_{\max} LF - R_b - D_b - Z_b)P_t - \frac{s v P_t}{d}\right) \quad (3)$$

Where:

Δt = change in time (d)

P_t = benthic algae biomass (mg/m^2) at current time step

$P_{t+\Delta t}$ = benthic algae biomass (mg/m^2) at next time step

μ_{max} = maximum algal growth rate (1/d)

LF = limiting factor (unitless)

R_b = algal respiration rate (1/d)

D_b = algal predatory and non-predatory mortality (1/d)

Z_b = algal grazing mortality (1/d)

s = scouring factor (unitless)

ν = water velocity (m/d)

d = water depth (m)

However, both minimum and maximum algal biomass values were employed to represent the restrictions of the physical world for algae growth that are not represented by the respiration, mortality, grazing rates or scour factor. Therefore, if Equation 3 produced an amount of algae that was either larger than the set maximum or smaller than the set minimum, the model substituted the maximum or minimum, respectively.

Scouring of benthic algae

A component of the benthic algae biomass calculation is scouring. Scouring occurs when benthic algae is removed from the bed of the river due to the force of the water flowing above it. Scouring will increase with the velocity of the water. Therefore, when the biomass equation was rewritten for an area-based calculation, not a volumetric calculation, the water velocity was retained in the scouring equation. Also a scouring factor was added, represented the percentage of benthic algae that is removed from the river bed by the water flow.

Limiting Factors

To more accurately calculate the algae biomass, the maximum growth rate for algae, taken from the literature, must be tempered with limiting factors. These factors take into account the limitations on growth due to available light, available nutrients, and the effect of temperature on algae growth. The apparent growth rate is represented as shown in Equation (4).

$$\mu = \mu_{max} f(T) f(L,P,N,C,Si) \tag{4}$$

μ = phytoplankton growth rate (1/day)

μ_{max} = maximum phytoplankton growth rate (1/day)

$f(T)$ = temperature correction (unitless)

L = light limitation (unitless)

P = phosphorous limitation (unitless)

N = nitrogen limitation (unitless)

C = carbon limitation (unitless)

Si = silica limitation (unitless)

The function $f(L,P,N,C,Si)$ represents one of several methods used to characterize algal growth limitation due to several interacting factors, and will be outlined further below.

Temperature

A Van't Hoff Arrhenius formulation is used to accommodate growth rates at temperatures other than 20°C.

$$G_T = G_{max}(\theta)^{T-20} \tag{5}$$

Where:

G_T	= temperature adjusted growth rate (1/day)
G_{max}	= maximum growth rate at 20°C (1/day)
θ	= temperature adjustment factor (1.047)
T	= ambient water temperature (°C)

Light

Algae utilize available underwater light for photosynthesis and the subsequent metabolic processes and cell growth. Solar radiation can be used to represent available light.

Light limitation fraction can be represented as

$$f(L) = (1-GSF)I/(K_L + I) \tag{6}$$

Where:

$f(L)$	= light limitation fraction ($0 \leq f(L) \leq 1$)
I	= light intensity (W/m^2, solar radiation)
GSF	= global shade factor, unitless
K_L	= light half saturation constant (8.37 W/m^2)

For the Shasta River algae model, both a global shade factor and hourly solar radiation were used to determine hourly light limitation fraction. If the global shade factor was equal to zero, there was no shade. If the global shade fraction was equal to one, there was complete darkness. When combined with the measured hourly solar radiation, the global shade fraction is a very flexible tool for evaluating the effects of cloud cover or vegetative cover on algal biomass. Because hourly solar radiation data was used, at night and in the early morning $f(L)$ equals 0.

Nutrients

The nutrients represented in the model include phosphorous, nitrogen, and silica. Carbon is assumed to be plentiful in the river system and does not limit algal production. Nutrient concentrations for the Shasta River algae model can be input as hourly concentrations, and therefore the limiting factors for each nutrient are calculated hourly as well. The equations for calculating the limitations of growth due to nutrients are as follows.

$$f(P) = \frac{PO_4^{3-}}{K_P + PO_4^{3-}} \quad (7)$$

$$f(N) = \frac{(NH_4^+ + NO_3^-)}{K_N + (NH_4^+ + NO_3^-)} \quad (8)$$

$$f(Si) = \frac{Si}{K_{Si} + Si} \quad (9)$$

Where:

$f(P)$	= phosphorous limitation fractions (unitless)
PO_4^{3-}	= orthophosphate concentration (mg/l)
K_P	= phosphorous half saturation constant (mg/l)
$f(N)$	= nitrogen limitation fractions (unitless)
NH_4^+	= ammonia concentration (mg/l)
NO_3^-	= nitrate concentration (mg/l)
K_N	= nitrogen half saturation constant (mg/l)
$f(Si)$	= silica limitation fractions (unitless)
S_i	= silica concentration (mg/l)
K_{Si}	= silica half saturation constant (mg/l)

Combined Limiting Factors – f(L,P,N,Si)

The combined limiting factors for light and nutrients can be determined using several methods, including multiplicative, minimum, harmonic mean, and arithmetic mean.

Multiplicative

$$f(L,P,N,Si) = f(L) \cdot f(P) \cdot f(N) \cdot f(Si) \quad (10)$$

Minimum

$$f(L,P,N,Si) = minimum[f(L), f(P), f(N), f(Si)] \quad (11)$$

Harmonic Mean

$$f(L,P,N,Si) = \frac{n}{\left(\frac{1}{f(L)} + \frac{1}{f(P)} + \frac{1}{f(N)} + \frac{1}{f(Si)}\right)} \qquad (12)$$

Arithmetic Mean

$$f(L,P,N,Si) = \frac{\left(f(L) + f(P) + f(N) + f(Si)\right)}{4} \qquad (13)$$

Comparison of these methods illustrates that the multiplicative formulation is the most limiting, while the arithmetic mean is the least limiting. However, because the light limiting factor can be equal to zero during the night and the early morning, only the multiplicative and minimum methods represent the correct combined limiting factors when using hourly solar radiation data. For the Shasta River algae model, the minimum combined limiting factor method was used.

Model Implementation

Presented in Table 1 are typical values for parameters necessary for the benthic algae model.

Table 1. Typical parameter values necessary for algal mass balance

	Parameter Values[a]							
	Growth Rate (1/d)	Respiration (1/d)	Mortality (1/d)	Grazing (1/d)	K_L (W/m^2 d)	K_N (mg/l)	K_P (mg/l)	K_{Si} (mg/l)
Total Phytoplankton	1.0-3.0	0.05 to 0.15	0.003 to 0.17	0.01 to 0.07	8.37 to 25.12	0.01 to 0.40	0.0005 to 0.03	0.03 to 0.10

[a] Values represent predominately freshwater systems

Those values used to implement the Shasta River algae model are presented in Table 2. The hourly solar radiation data used in model implementation is 2000 solar radiation from Brazie Ranch (with small data gaps filled using linear interpolation and large data gaps filled using 2000 meteorological data from Klamath Falls (Oregon AgriMet station KFLO, supported by U.S. Bureau of Reclamation) solar radiation). The hourly water temperature data used 2000 water temperature data for the mouth of the Shasta River complied for the Klamath River modeling project. The light extinction coefficient was provided from existing Shasta River field data.

The travel time and reach dimensions were approximate estimates of typical Shasta River conditions. A rectangular cross-section shape was assumed for the fictitious reach. While the model is built to accommodate hourly flow and nutrient data, as the reach was fictitious, it was determined that constant flow (and therefore constant velocity in the reach) and constant nutrient concentrations would allow for a better understanding of the model's functions. The resulting algae biomass from model implementation is presented in Figure 1.

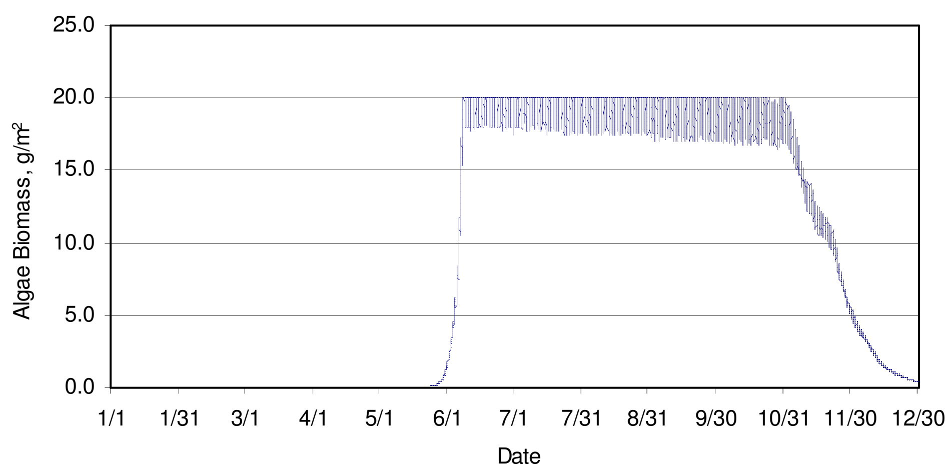


Figure 1. Algal biomass with model implementation parameters

Table 2. Model parameter values for implementation

Parameter	Model Value	Units
Time step	0.041667	day
Travel time of reach	0.042	day
Reach length, l	1609	meters
River width, w	9.1	meters
River depth, d	0.6	meters
River cross-sectional area, CS	13.9	m^2
Reach volume, V	22426.9	m^3
Reach flow in and flow out, Qin and Qout	538247	m^3/day
Reach bed area, A	7357.9	m^2
Reach velocity, vel	73.2	m/day
Initial bed algae biomass, P_i	0.001	g/m^2
Minimum bed algae biomass, P_{min}	0.1	g/m^2
maximum bed algae biomass, P_{max}	20	g/m^2
Solar radiation, SR	hourly	W/m^2
Global Shade Factor, GSF	0	-
Total inorganic nitrogen inflow concentration, $[TIN]_{in}$	0.2	mg/l
Phosphate inflow concentration, $[PO4]_{in}$	0.2	mg/l
Silica inflow concentration, $[Si]_{in}$	50	mg/l
Light half saturation coefficient, K_L	0.0009	Kcal/m^2s
Light extinction coefficient, Le	1.48	1/meter
Nitrogen half saturation coefficient, K_N	0.014	mg/l
Phosphate half saturation coefficient, K_P	0.003	mg/l
Silica half saturation coefficient, K_S	0.03	mg/l
Maximum growth rate, G	1.2	1/day
Respiration (and excretion) rate, R	0.14	1/day
Mortality rate, D	0.14	1/day
Grazing rate, Z	0.05	1/day
Algae settling rate, v	0	m/day
Scouring factor, s	0.00001	-
Theta, θ	1.040	-
Water Temperature, T	hourly	C
Reference water temperature, T_{ref}	20	C

Sensitivity Analysis

A test of sensitivity was performed on the model to determine what parameters, if any, to which the model is sensitive. The sensitivity analysis was restricted to nutrient half-saturation coefficients, nutrient concentrations, the light extinction coefficient, the depth of the river (changing the depth altered the flow rate in the model since the flow rate is determined from the dimensions of the river reach and the travel time), the maximum growth rate, the global shading factor, the initial algal biomass per area, and the maximum and minimum algal biomass per area. Based on the sensitivity analysis, there are several conclusions that can be drawn about the model.

The model is not sensitive to silica half-saturation constants or concentrations. The model is mildly sensitive to phosphate half-saturation constants and concentrations, and is sensitive to nitrogen half-saturation and concentrations.

For both phosphate and nitrogen, when the concentration of nutrient approached the half-saturation for that nutrient, the algal biomass was decreased, and vice versa, if the nutrient concentration retreated from the half-saturation constant, the algal biomass increased. Maintaining the modeling implementation nitrogen half-saturation constant of 0.014 mg/l, a nitrogen concentration of 0.02 mg/l (an order of magnitude lower than the model implementation value) created only 10% of the model implementation biomass. If the nitrogen concentration was lowered to equal the half-saturation concentration, essentially no algae was produced during the year. The same was true for lowering the phosphate concentration to equal the half-saturation constant. However, lowering the phosphate concentration one order of magnitude to 0.02 mg/l only lowered the biomass to 92% of the model implementation biomass. Increasing the nitrogen concentration by an order of magnitude or decreasing the half-saturation constant by an order of magnitude both increased the algal biomass to 104 % of the model implementation biomass. Increasing the phosphate concentration by an order of magnitude or decreasing the half-saturation constant by an order of magnitude both had no effect on the annual biomass. Increasing the half-saturation constant for phosphate produced the same 92% biomass as decreasing the phosphate concentration to 0.02 mg/l.

Combinations of increasing or decreasing all of the half-saturation or concentrations of nutrients together did affect the results in a none-additive manner. When the half-saturation constants were all lowered an order of magnitude, there was an increase in the biomass of 104%, but the annual cumulative biomass is slightly larger than when only the nitrogen half-saturation constant is lower. Increasing all of the half-saturation constants by an order of magnitude produced the same result as only increasing the nitrogen half-saturation constant.

All nutrient sensitivity results are presented in Table 3 and Figure 2 through Figure 7, and Figure 15 through Figure 32.

There was a linear relationship between the light extinction coefficient, Le, and the annual average algal biomass, P_{ave}. Increasing Le decreased P_{ave} slightly, but still well within the same order of magnitude, as shown in Table 3, Figure 8 , and Figure 33 through Figure 36. The yearly graphs show that increasing the Le slightly decreases the amount of algae produced in the latter portion of the growing season. The relationship between river depth, d, and P_{ave} was similar to the Le vs P_{ave} relationship as P_{ave} decreased with increasing d and the size of change in P_{ave} was not very large, as shown in Table 3 and Figure 9. Also, the same changes in the production at the end of the growing season occurred for increased d as they did for increased Le, as shown in Figure 37 through Figure 40. The similar relationships for Le and d were expected as the amount of light reaching the bottle of a river bed decreases with increases in either d or Le.

There was also a linear relationship between maximum algal growth rate, G, and P_{ave}. Increases in G produced increases in P_{ave}. However, incremental increases in G did not increase the order of magnitude of P_{ave}, as shown in Table 3 and Figure 10. As shown in Figure 41 through Figure 44, increasing G increased the length of the growing season by starting the algae bloom earlier in the year.

The global shade factor, GSF, decreased P when increased, but it did not indicate a linear relationship. Rather, it appeared that the decrease in P_{ave} was smaller with increased GSF until GSF reached 0.5 (or 50% shade) and then the increases in GSF produced larger decreases in P_{ave} until there is approximately 60% of the model implementation biomass when GSF equals 0.9. The sensitivity analysis results for varying GSF can be seen Table 3 and Figure 11.As seen in Figure 45 through Figure 48, increasing GSF shortened the length of the growing season by both delaying the start of the algae bloom and curtailing the period of time in which the algae would flourish until there is no growing season for a GSF of 0.9.

Increases in the minimum algal biomass per area, P_{min}, produced very small increases in P_{ave}. There was little change to P_{ave} even when P_{min} was increased by an order of magnitude. This indicates that this model implementation rarely produced an algal biomass per area of less than 1 g/m^2. Sensitivity analysis results for P_{min} are presented in Table 3 and Figure 12. There were no overall seasonal changes in the timing of growth or the length of the growing season, as presented in Figure 49 through Figure 52.

Increases in the maximum algae biomass per area, P_{max} created large increases of P_{ave} in a linear relationship to each other. The sensitivity of P_{max} was tested to the large range presented in Table 3 to determine if there was a maximum algal biomass per area that the model would achieve on its own. The value for P_{max} that was found to allow the model to always use the calculated algal biomass per area was very large. The large value underlines both the inherent problems in modeling a processes as complex as algal growth in a river as well as the necessity of using parameters such as P_{max} and P_{min} in assisting the model to calculate results feasible to the physical world. Sensitivity analysis results for P_{max} are presented in Table 3 and Figure 13. Illustrated in Figure 53 through Figure 57 is the change in both maximum algal biomass per area and the start of the growing season. As P_{max} increased, the start of the growing season was delayed very slightly, until, with the largest value of P_{max} shown, the growth season has been delayed by several months but ends normally, so is quite short.

Increasing the initial algal biomass per area, P_i, produced small increases in P_{ave}. As can be seen in Table 3, increasing P_i by three orders of magnitude only increased P_{ave} to 110.3% of the implementation value. Further investigation into P_i and its effect on P_{ave}

showed that the values of P asymptotically approached 10.89 g/m^2 until P_i reached 20, and then remained a constant 10.89 g/m^2 with further increases in P_i. However, this maximum value is directly related to the maximum algal biomass per area, P_{max}, which is specified by the user of the model, in this case specified to be 20 g /m2. Changing P_{max} would alter both the constant maximum P_{ave} that is asymptotically approached as well as the maximum P_i at which the constant P_{ave} would be achieved. Graphically, increases in P_i produced both an unstable algal population in the middle of winter which decreases to normal levels until the start of the regular growing season, and a hastening of the start of the growing season. Sensitivity analysis results for P_i are presented in Table 3, Figure 14, and Figure 58 through Figure 63.

Table 3. Annual total and annual average algae biomass sensitivity analysis results

Varied Parameter(s)	Parameter(s) Value	Units	Annual Total Biomass	Annual Ave Biomass	% Baseline
None (Baseline Condition)	Implementation values	-	77913	8.87	100%
K_N	0.0014	mg/l	80976	9.22	104%
	0.14		7564	0.86	10%
K_P	0.0003	mg/l	77913	8.87	100%
	0.03		71489	8.14	92%
K_{Si}	0.003	mg/l	77913	8.87	100%
	0.3		77913	8.87	100%
K_N, K_P, K_{Si}	0.0014, 0.0003, 0.003	mg/l	81010	9.22	104%
	0.14, 0.03, 0.3		7564	0.86	10%
$[TIN]_{in}$	0.014	mg/l	1	0.00012	0.0014%
	0.02		7564	0.86	10%
	2		80976	9.22	104%
$[PO4]_{in}$	0.003	mg/l	1	0.00012	0.0014%
	0.02		71489	8.14	92%
	2		77913	8.87	100%
$[Si]_{in}$	5	mg/l	77913	8.87	100%
	500		77913	8.87	100%
$[TIN]_{in}$, $[PO4]_{in}$, $[Si]_{in}$	0.02, 0.02, 5.0	mg/l	7564	0.86	10%
	2.0, 2.0, 500.0		81010	9.22	104%
Le	1.40	1/m	78390	8.92	101%
	1.44		78149	8.90	100%
	1.52		77683	8.84	100%
	1.56		77454	8.82	99%
d	0.15 (0.5)	m (ft)	88277	10.05	113%
	0.31 (1.0)		84307	9.60	108%
	0.92 (3.0)		73433	8.36	94%
	1.22 (4.0)		68195	7.76	87.5%
G	1.0	1/day	55527	6.32	71.3%
	1.1		67727	7.71	86.9%
	1.3		88193	10.04	113.2%
	1.4		95429	10.86	122.4%
GSF	0.1	-	76926	8.76	98.8%
	0.5		70736	8.05	90.8%
	0.7		64543	7.35	82.9%
	0.9		45184	5.14	57.9%
P_{min}	0.0	g/m^2	77590	8.83	99.6%
	0.2		78841	8.98	101.2%
	0.5		80039	9.11	102.7%
	1.0		80820	9.20	103.7%
P_{max}	30	g/m^2	116100	13.22	149.0%
	40		154114	17.54	197.7%
	50		191943	21.85	246.3%
	100		379504	43.20	487.0%
	1.00E+27		4.34E+22	4.94E+18	5.57E+17
P_i	0	g/m^2	77913	8.87	100.0%
	0.002		78485	8.94	100.8%
	0.005		79717	9.08	102.4%
	0.010		80535	9.17	103.4%
	0.100		83189	9.47	106.8%
	1.000		85953	9.78	110.3%

Parameter variation and Annual Average Algal Biomass Graphs

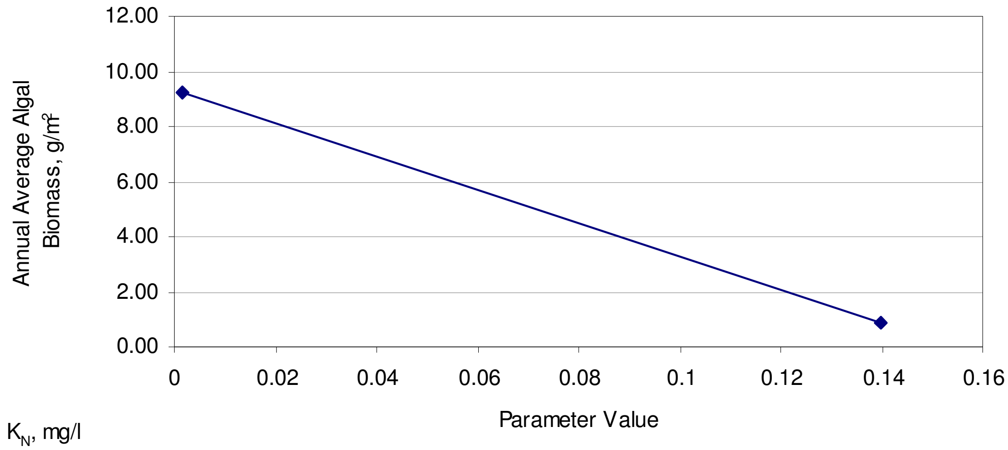


Figure 2. Annual average algal biomass when K_N was varied.

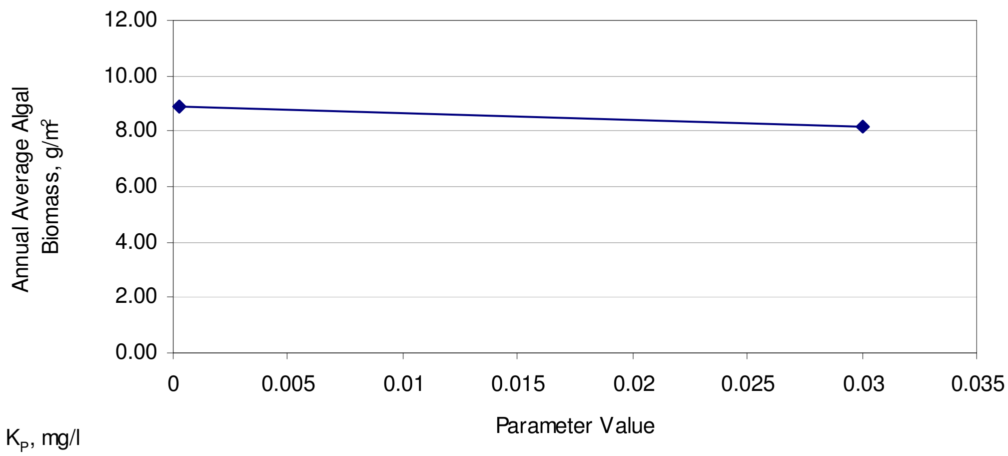


Figure 3. Annual average algal biomass when K_P was varied.

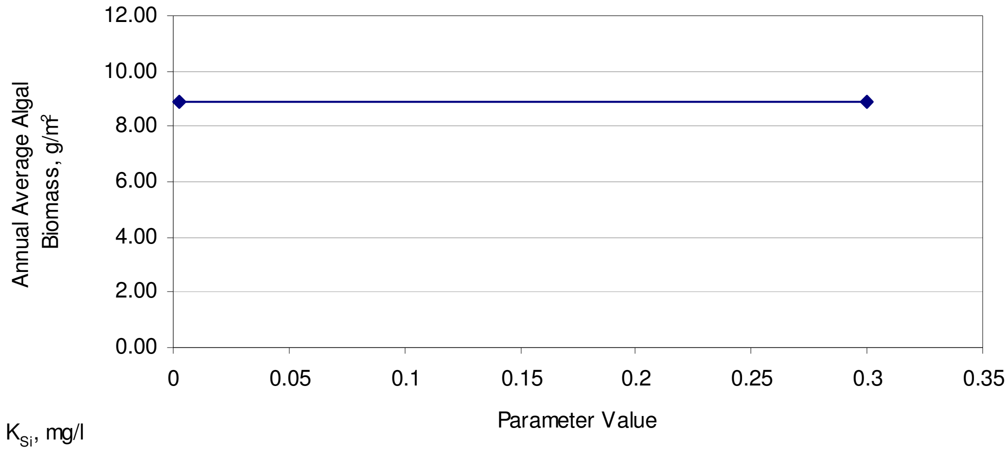


Figure 4. Annual average algal biomass when K_{Si} was varied.

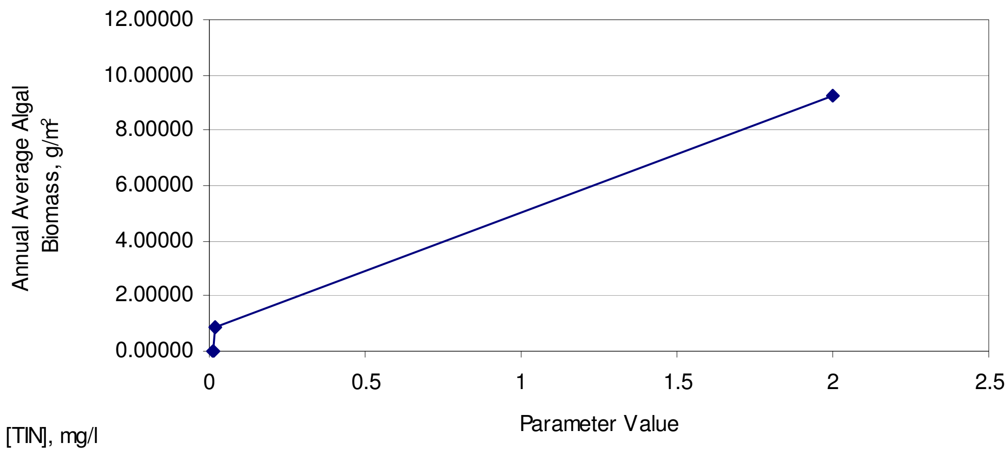


Figure 5. Annual average algal biomass when [TIN] was varied.

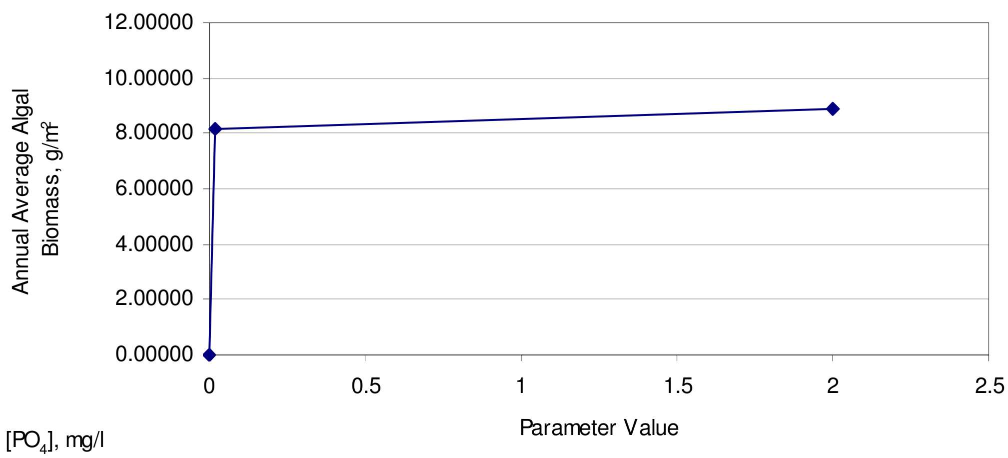
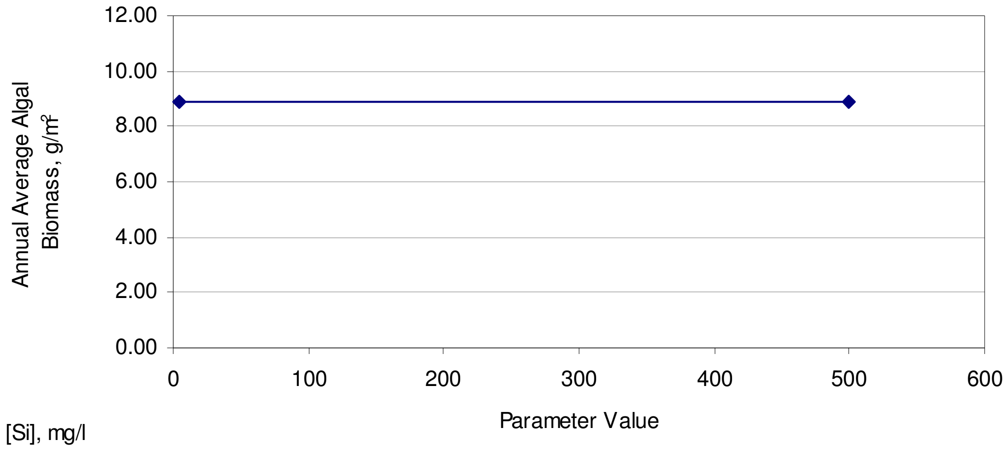
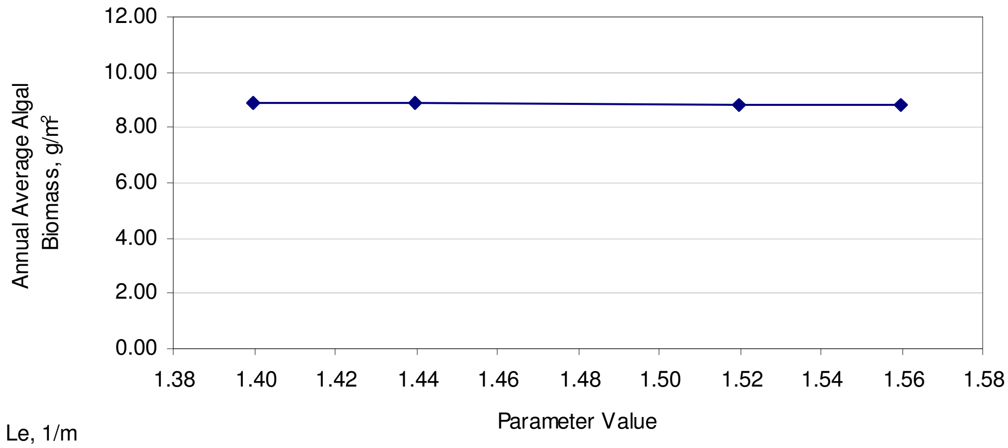


Figure 6. Annual average algal biomass when [PO_4] was varied.



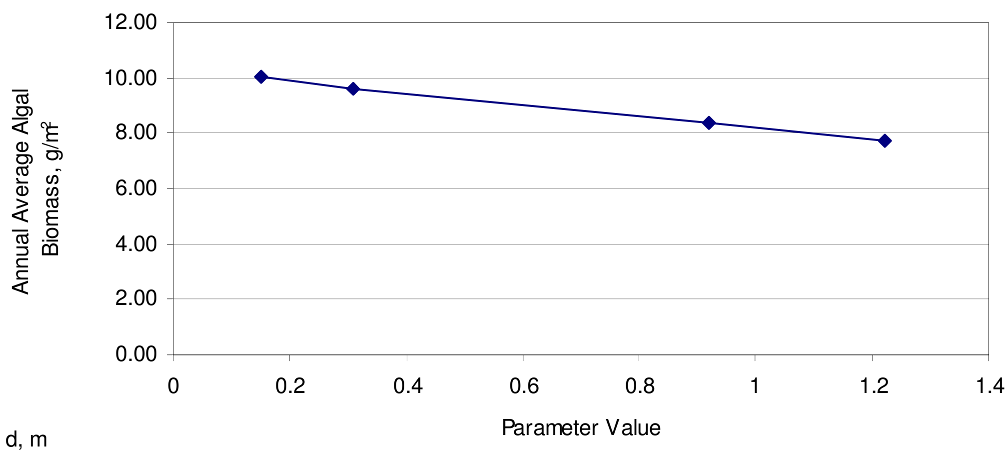
[Si], mg/l

Figure 7. Annual average algal biomass when [Si] was varied.



Le, 1/m

Figure 8. Annual average algal biomass when Le was varied.



d, m

Figure 9. Annual average algal biomass when d was varied.

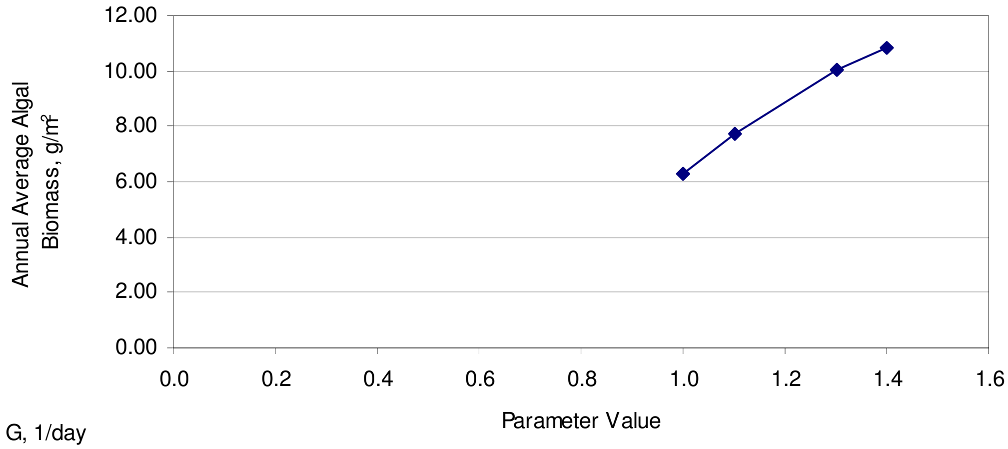


Figure 10. Annual average algal biomass when G was varied.

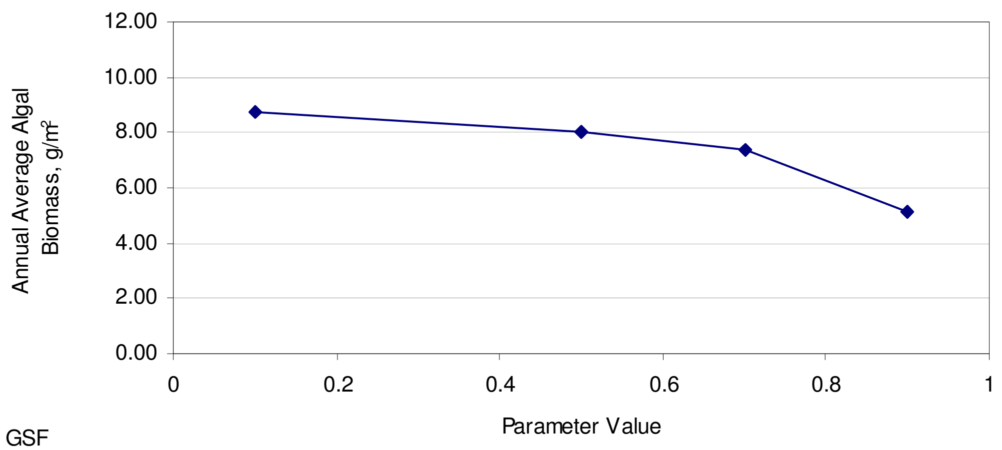


Figure 11. Annual average algal biomass when GSF was varied.

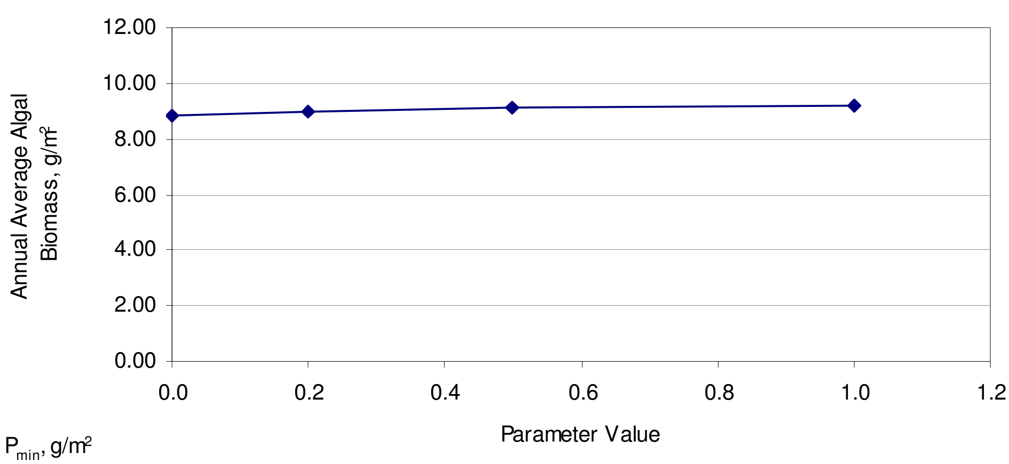
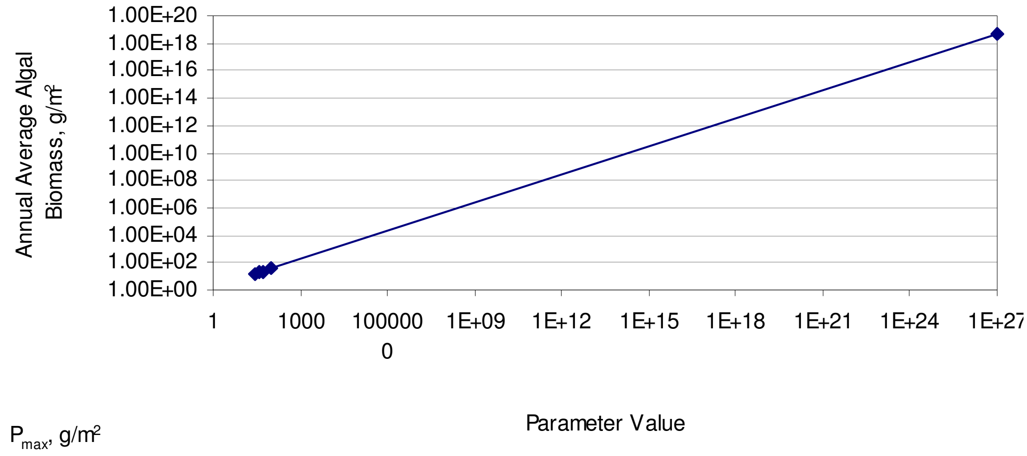
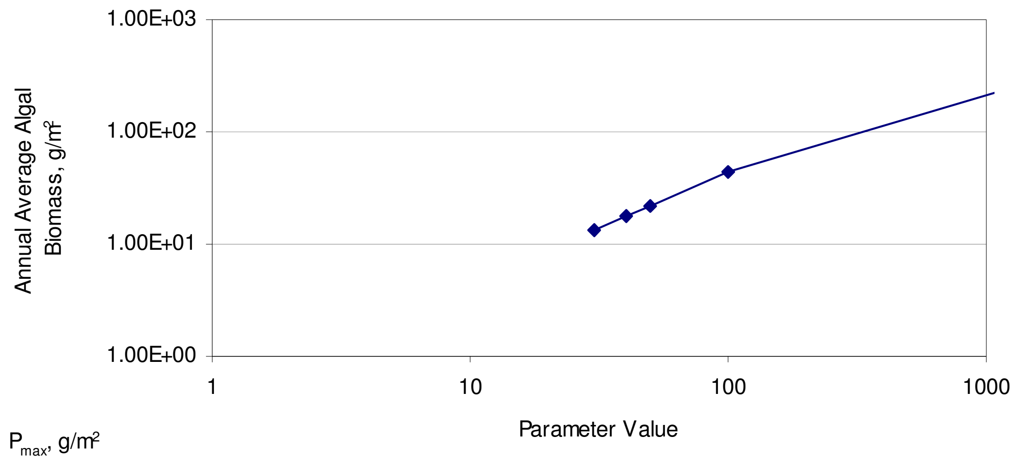


Figure 12. Annual average algal biomass when P_{Min} was varied.



(a)



(b)

Figure 13. Annual average algal biomass when P_{Max} was varied: (a) all values of P_{Max}; (b) smaller values of P_{Max}

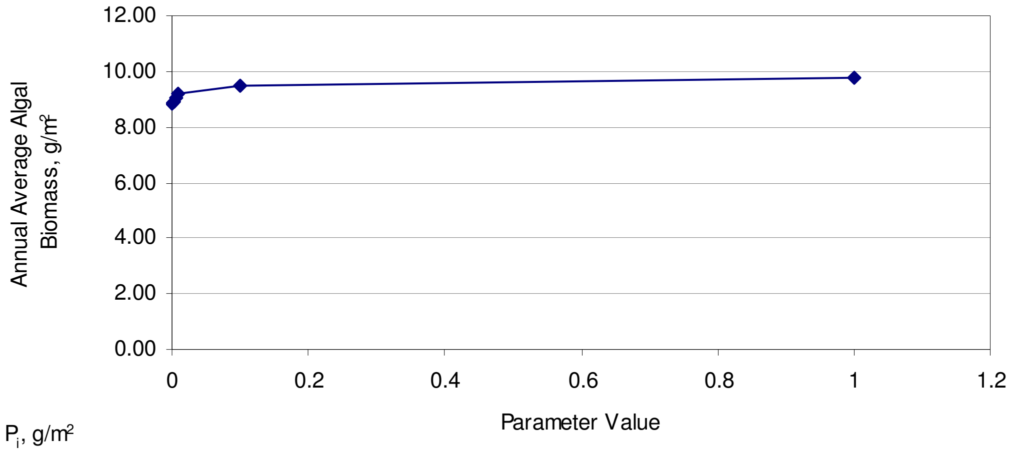


Figure 14. Annual average algal biomass when P_i was varied.

Algal Biomass graphical results for sensitivity analysis

Altering nutrient half-saturation coefficients

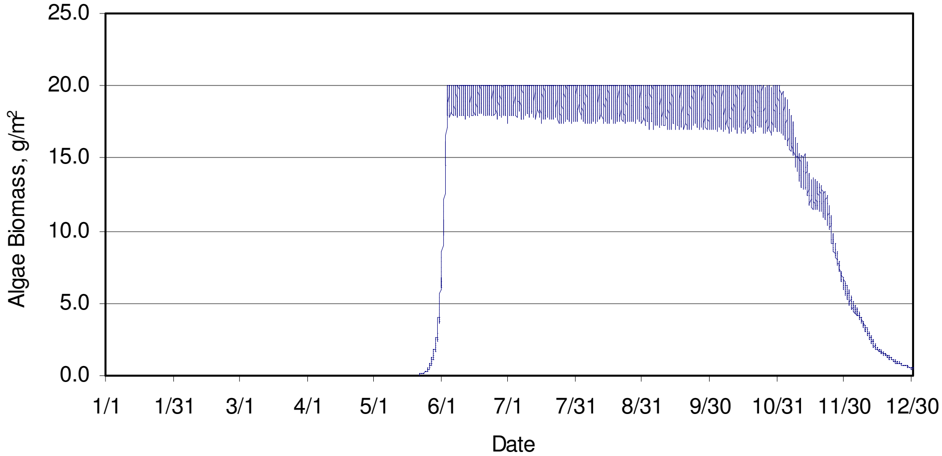


Figure 15. Algal biomass with Nitrogen half saturation coefficient equal to 0.0014 mg/l

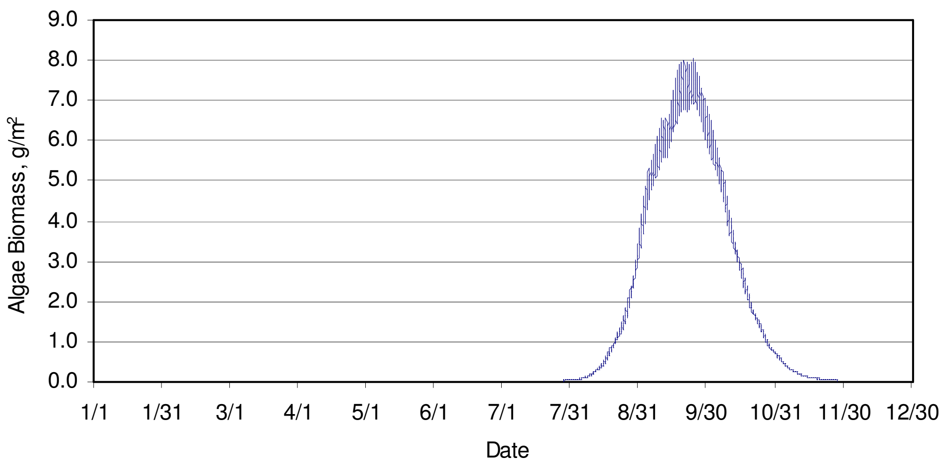


Figure 16. Algal biomass with nitrogen half saturation coefficient equal to 0.14 mg/l

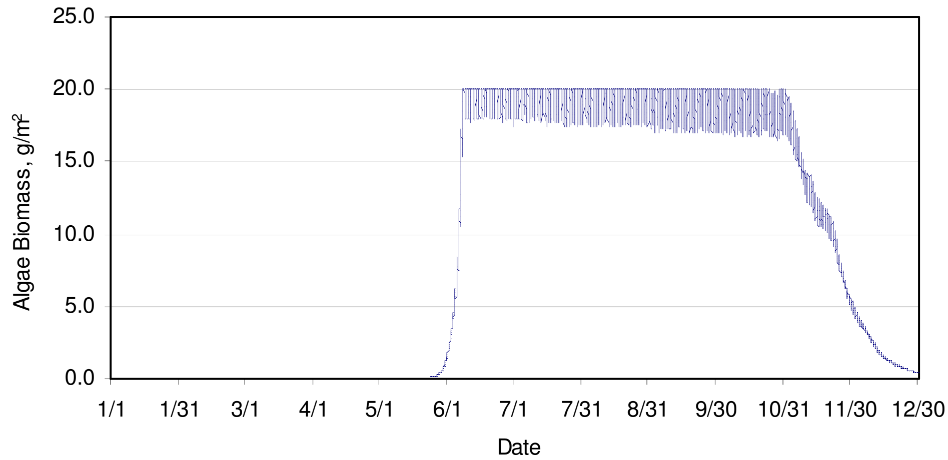


Figure 17. Algal biomass with phosphorus half saturation coefficient equal to 0.0003 mg/l

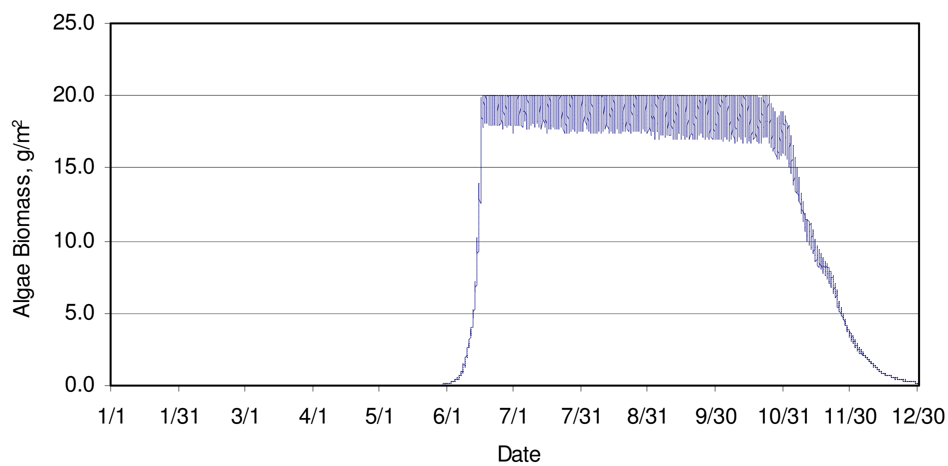


Figure 18. Algal biomass with phosphorus half saturation coefficient equal to 0.03 mg/l

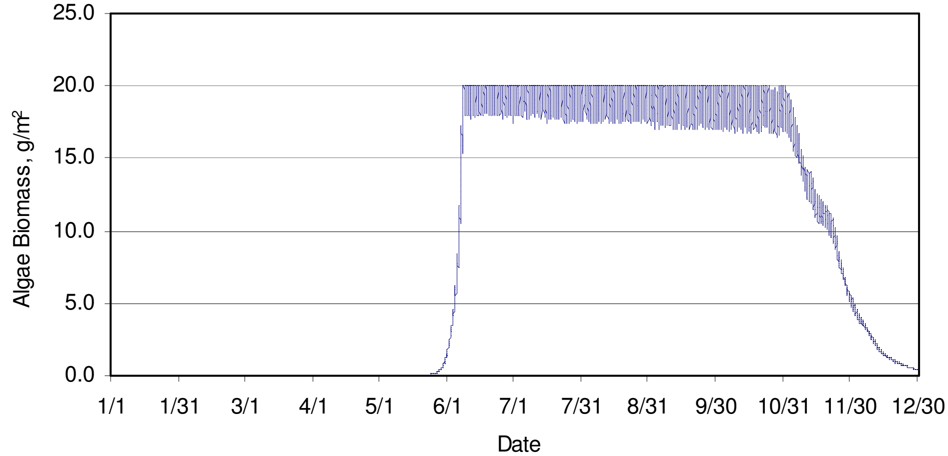


Figure 19. Algal biomass with silica half saturation coefficient equal to 0.003 mg/l

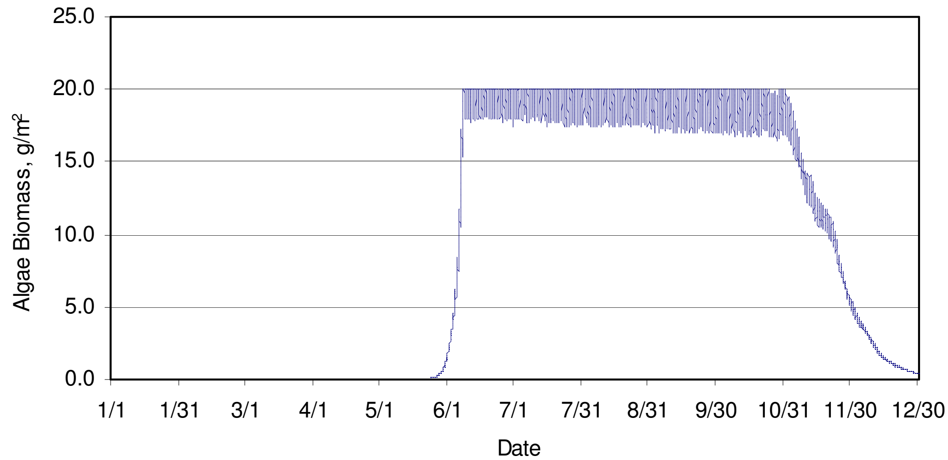


Figure 20. Algal biomass with silica half saturation coefficient equal to 0.3 mg/l

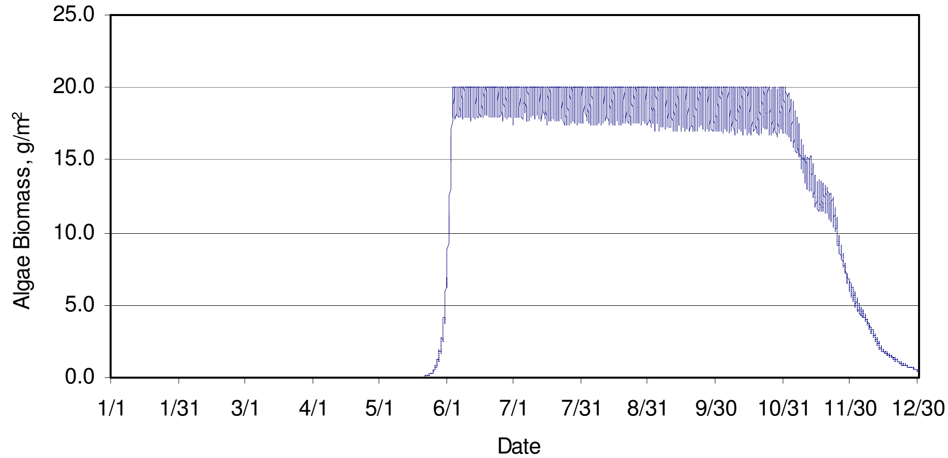


Figure 21. Algal biomass with K_N equal to 0.0014 mg/l, K_P equal to 0.0003 mg/l and K_S equal to 0.003 mg/l

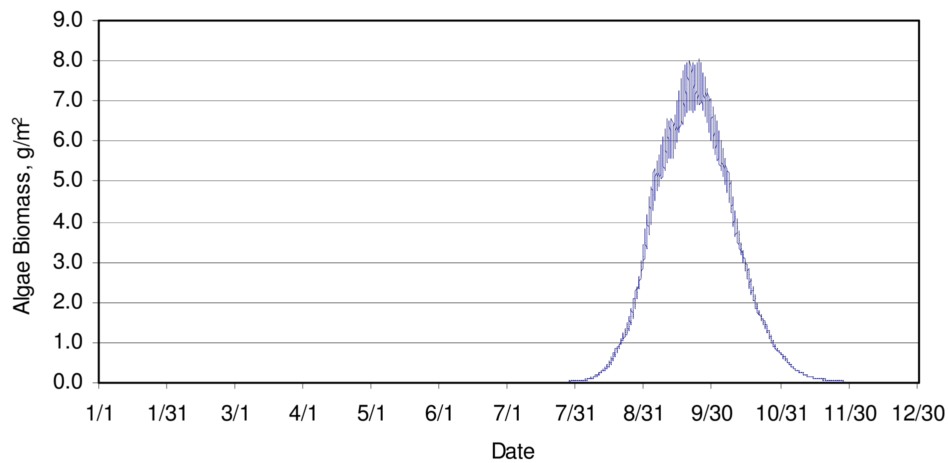


Figure 22. Algal biomass with K_N equal to 0.14 mg/l, K_P equal to 0.03mg/l and K_S equal to 0.3 mg/l

Altering nutrient concentrations

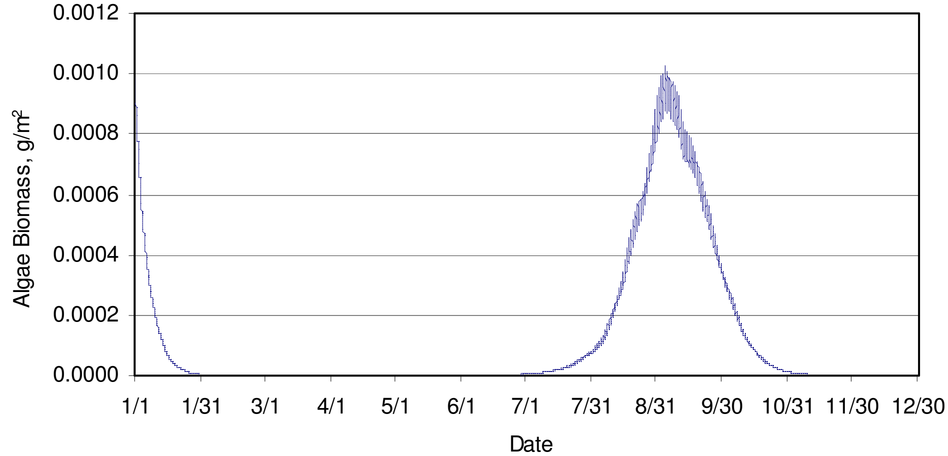


Figure 23. Algal biomass with total inorganic nitrogen concentration equal to 0.014 mg/l

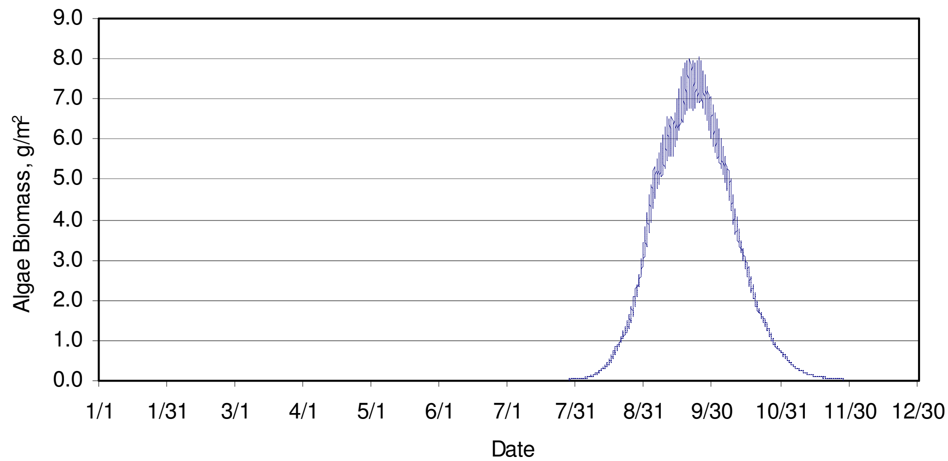


Figure 24. Algal biomass with total inorganic nitrogen concentration equal to 0.02 mg/l

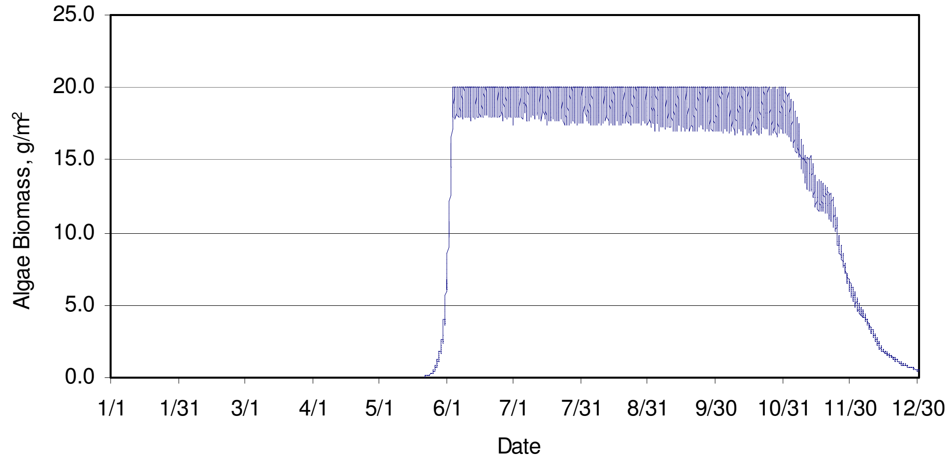


Figure 25. Algal biomass with total inorganic nitrogen concentration equal to 2.0 mg/l

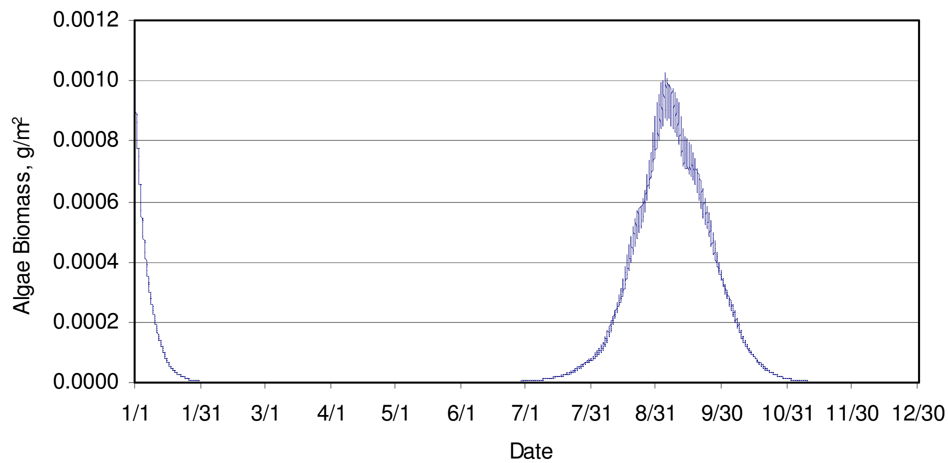


Figure 26. Algal biomass with phosphate concentration equal to 0.003 mg/l

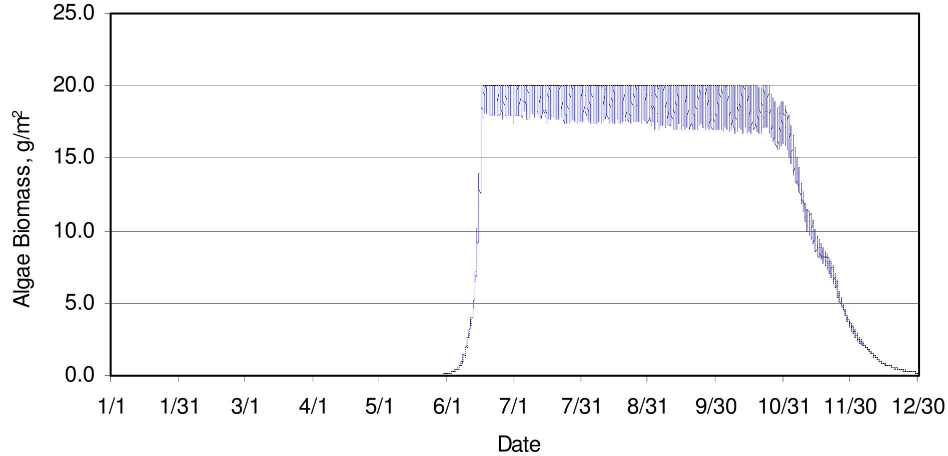


Figure 27. Algal biomass with phosphate concentration equal to 0.02 mg/l

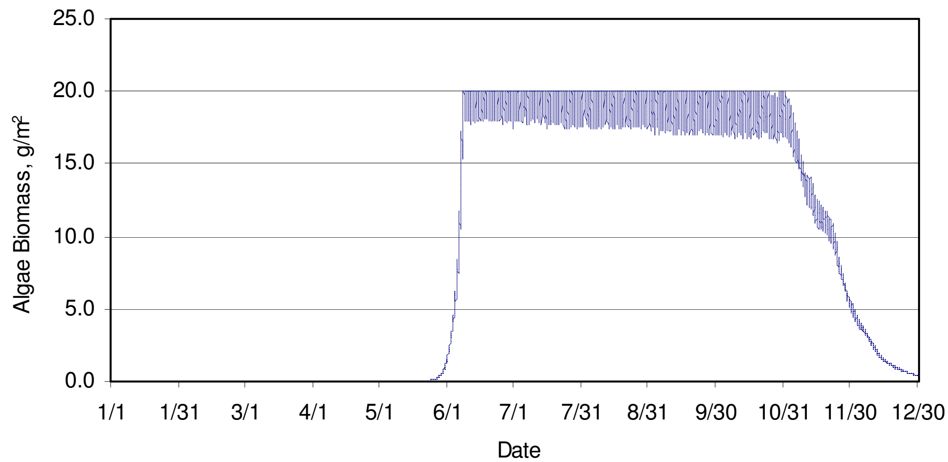


Figure 28. Algal biomass with phosphate concentration equal to 2.0 mg/l

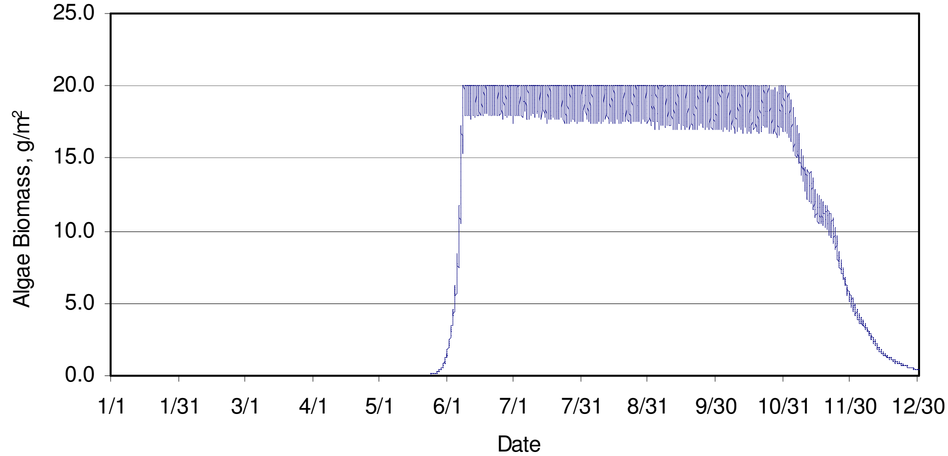


Figure 29. Algal biomass with silica concentration equal to 5.0 mg/l

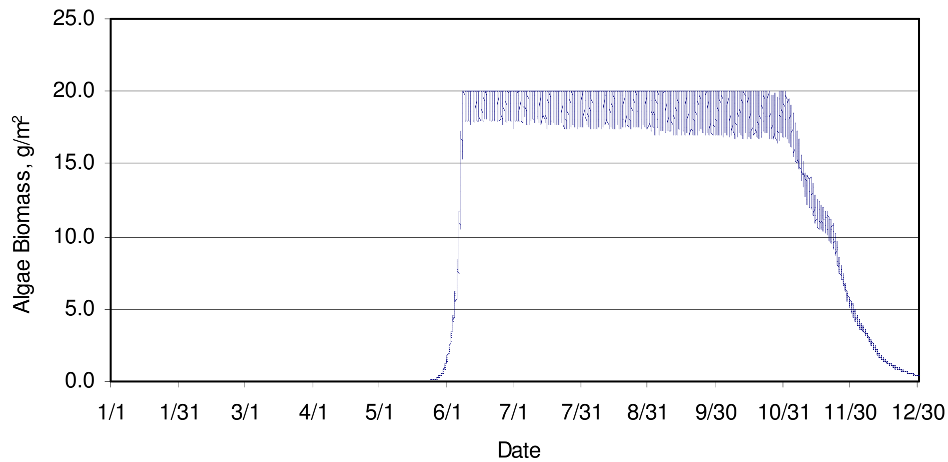


Figure 30. Algal biomass with silica concentration equal to 500.0 mg/l

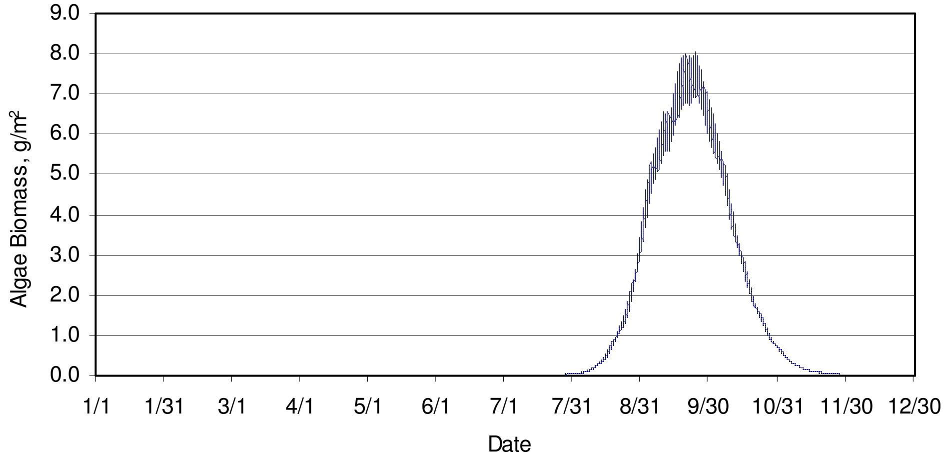


Figure 31. Algal biomass with TIN concentration equal to 0.02 mg/l, phosphate concentration equal to 0.02 mg/l, and silica concentration equal to 5.0 mg/l

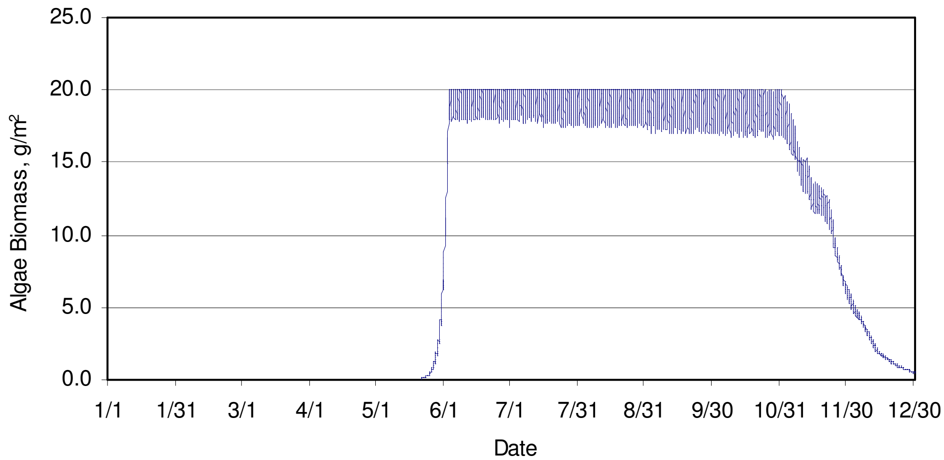


Figure 32. Algal biomass with TIN concentration equal to 2.0 mg/l, phosphate concentration equal to 2.0 mg/l, and silica concentration equal to 500.0 mg/l

Altering the light extinction coefficient

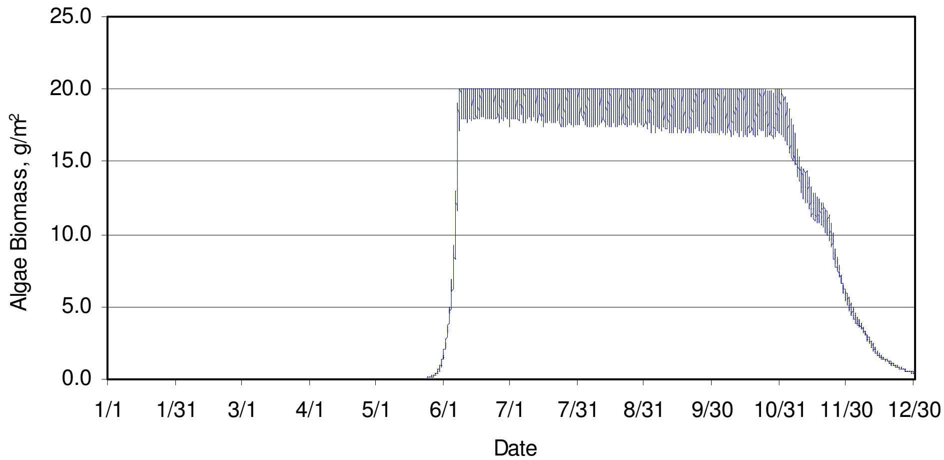


Figure 33. Algal biomass with light extinction coefficient equal to 1.40

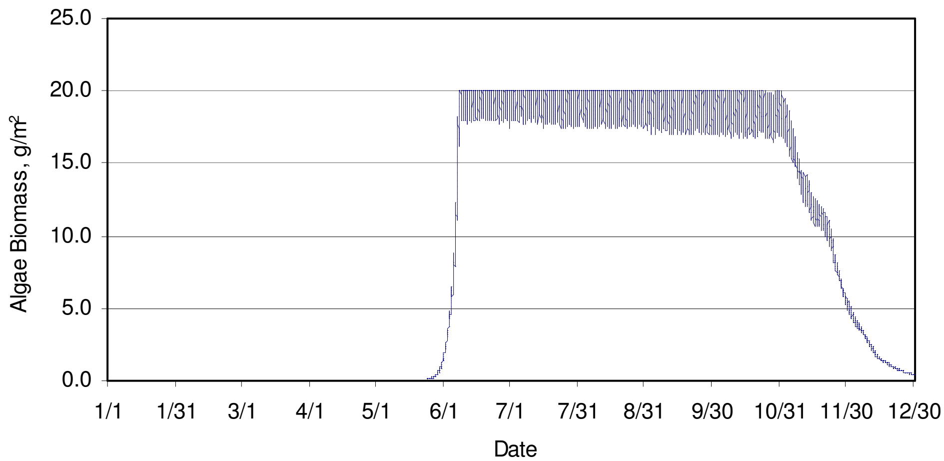


Figure 34. Algal biomass with light extinction coefficient equal to 1.44

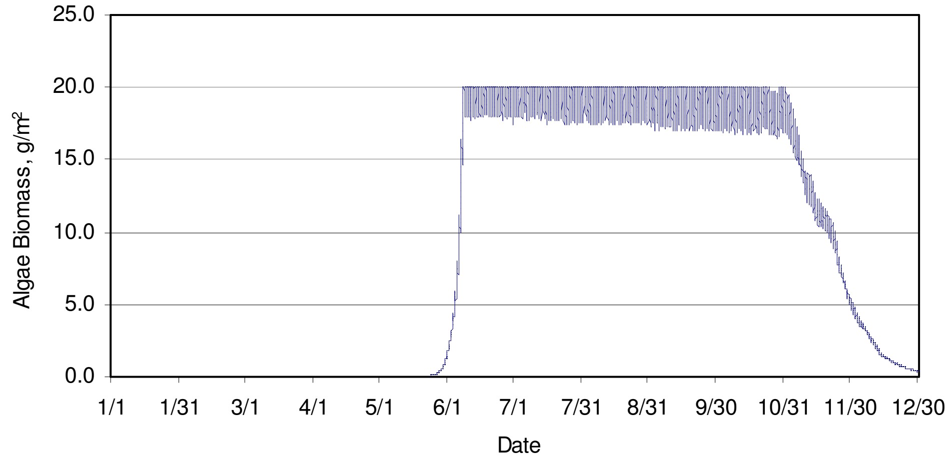


Figure 35. Algal biomass with light extinction coefficient equal to 1.52

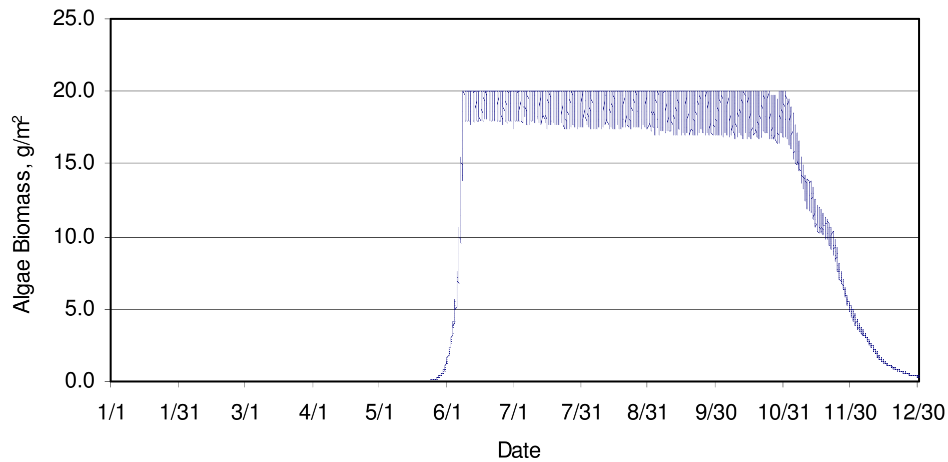


Figure 36. Algal biomass with light extinction coefficient equal to 1.56

Altering depth

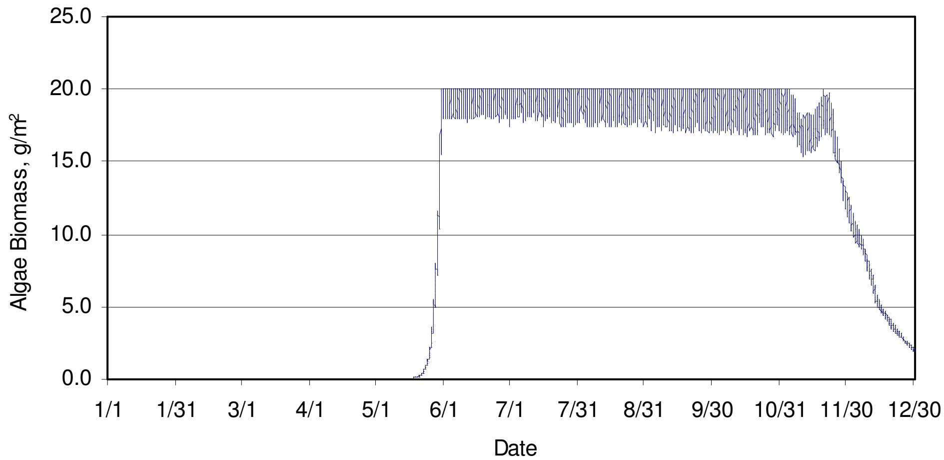


Figure 37. Algal biomass with depth equal to 0.2 meters

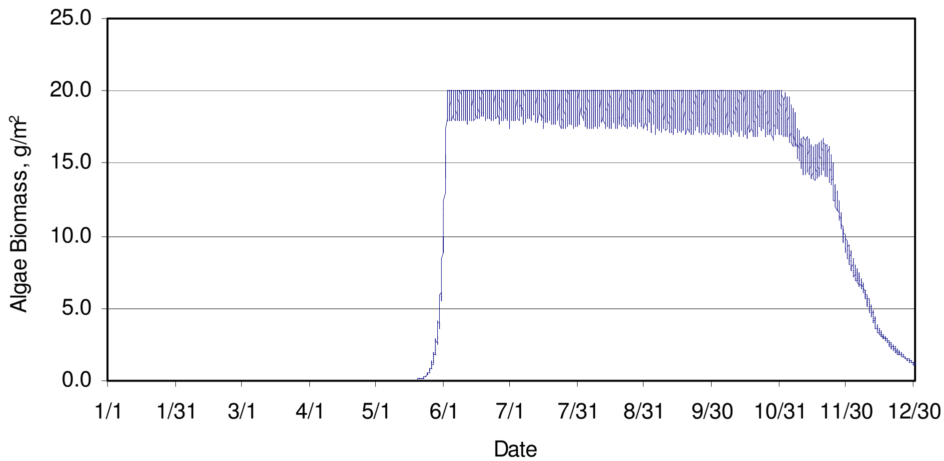


Figure 38. Algal biomass with depth equal to 0.3 meters

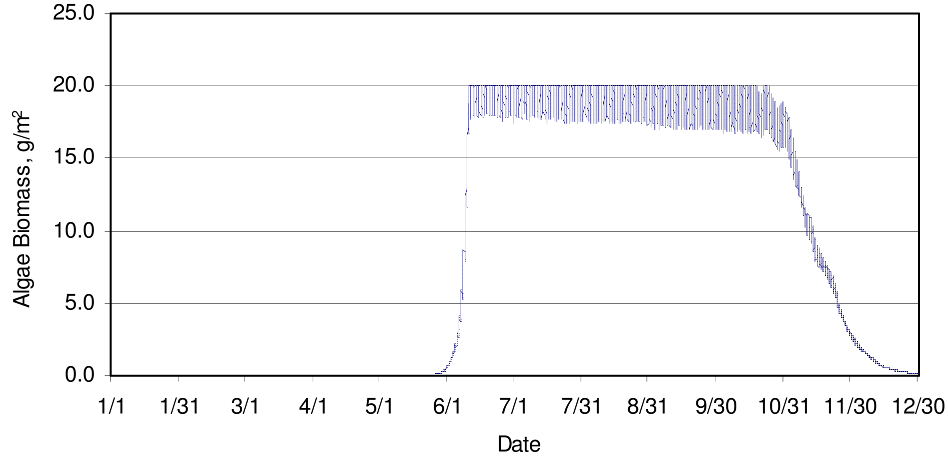


Figure 39. Algal biomass with depth equal to 0.9 meters

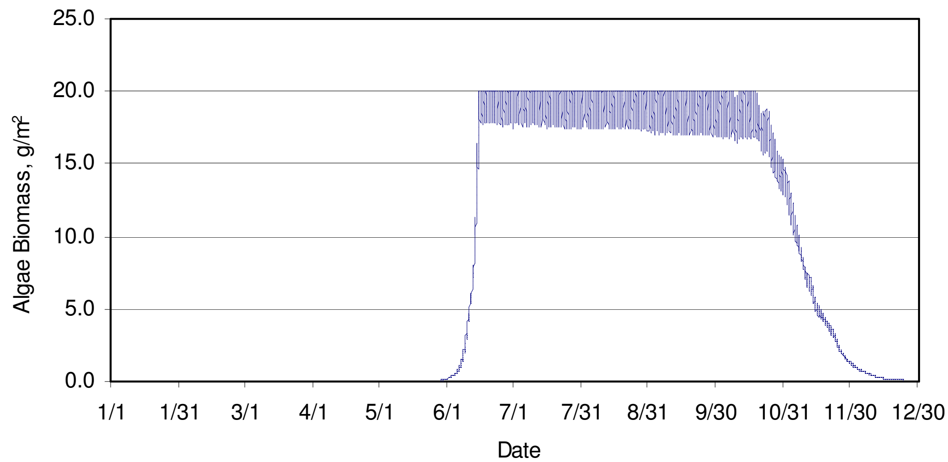


Figure 40. Algal biomass with depth equal to 1.2 meters

Altering maximum algal growth rate

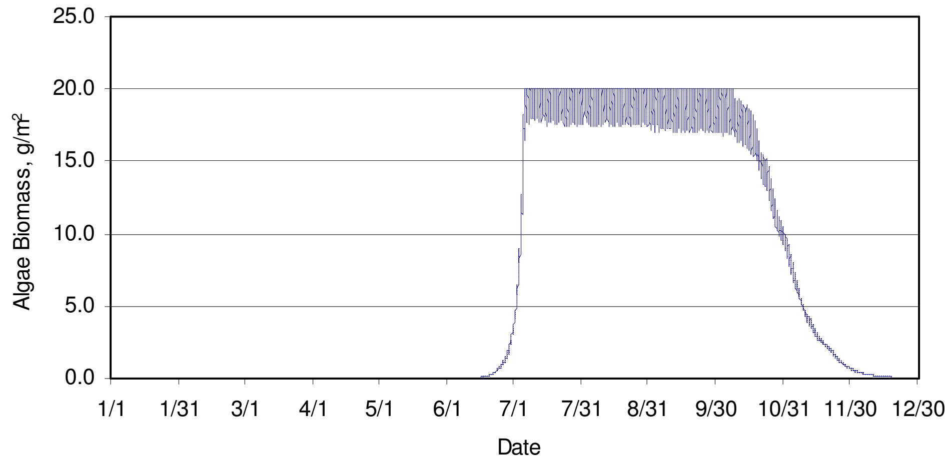


Figure 41. Algal biomass with maximum algal growth rate equal to 1.0 1/day

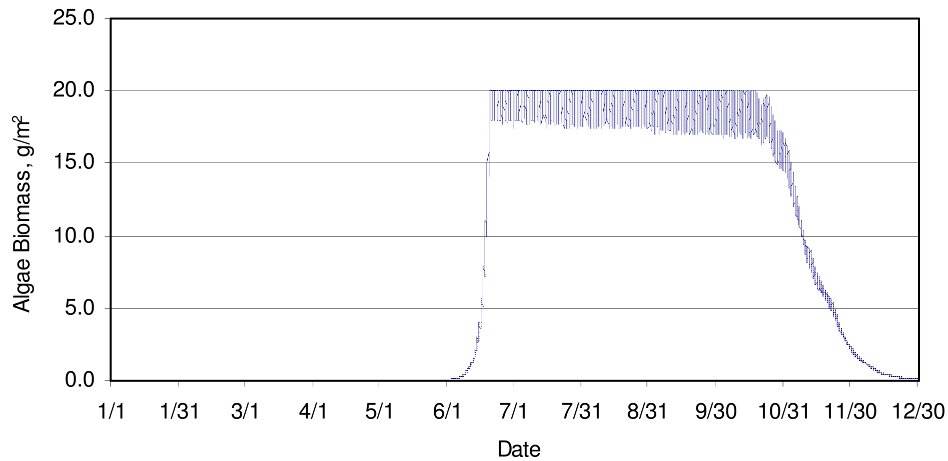


Figure 42. Algal biomass with maximum algal growth rate equal to 1.1 1/day

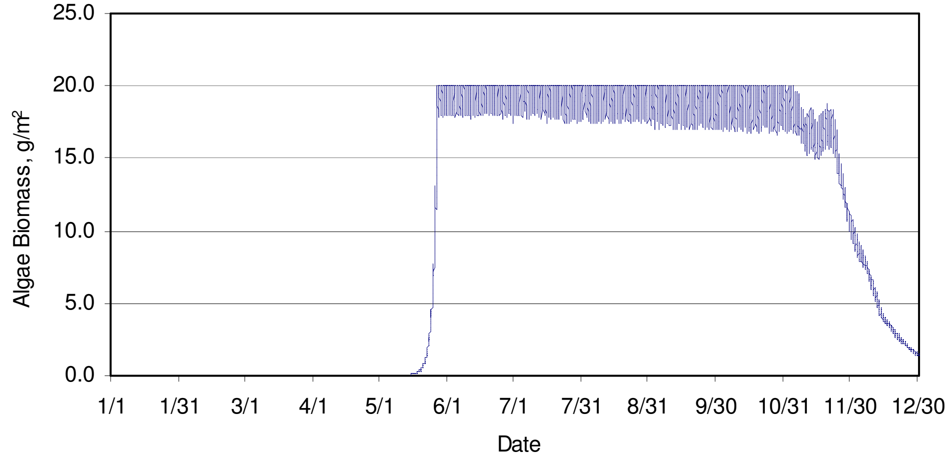


Figure 43. Algal biomass with maximum algal growth rate equal to 1.3 1/day

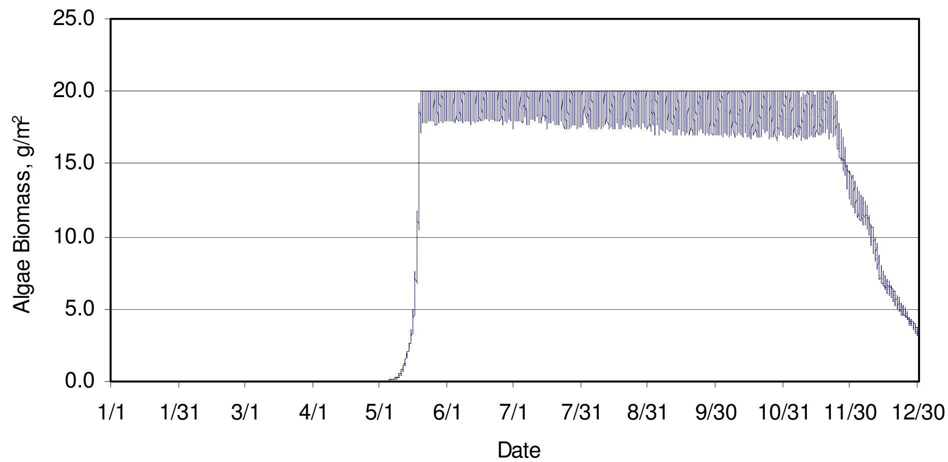


Figure 44. Algal biomass with maximum algal growth rate equal to 1.4 1/day

Altering Global Shade Factor

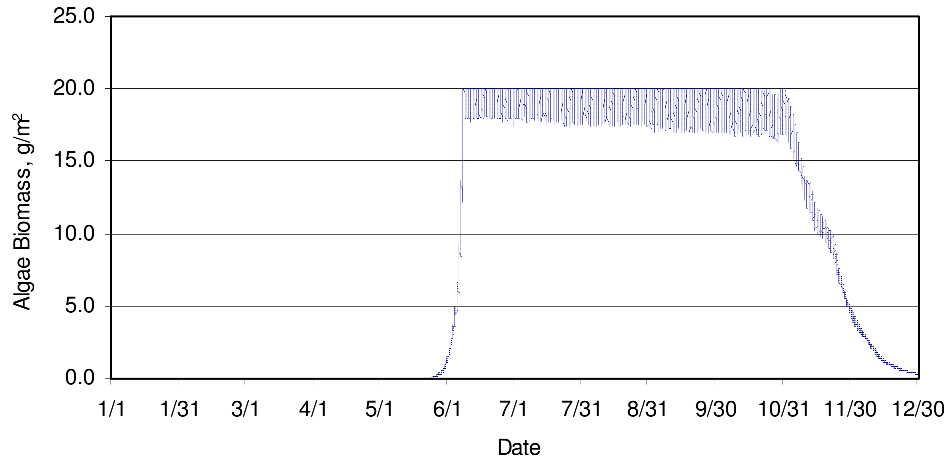


Figure 45. Algal biomass with global shade factor equal to 0.1

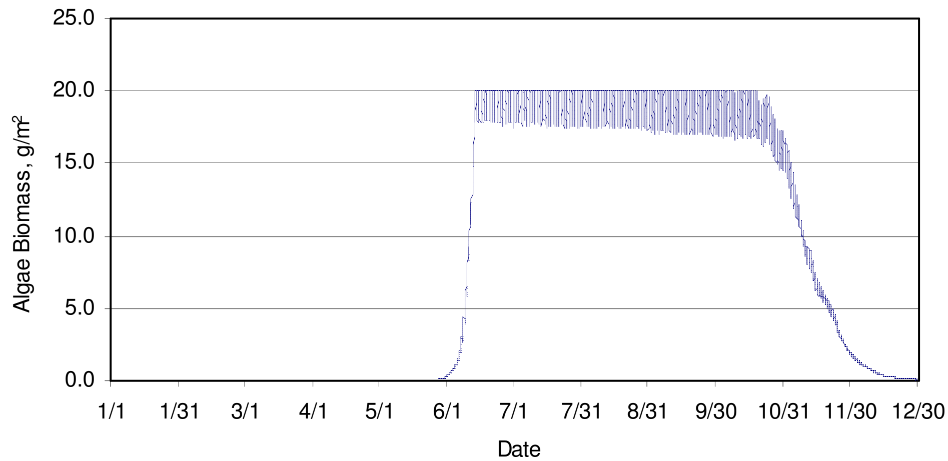


Figure 46. Algal biomass with global shade factor equal to 0.5

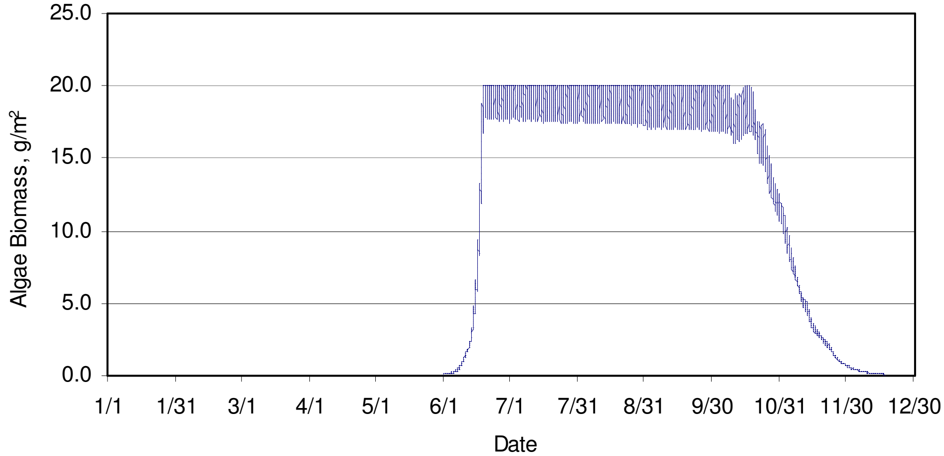


Figure 47. Algal biomass with global shade factor equal to 0.7

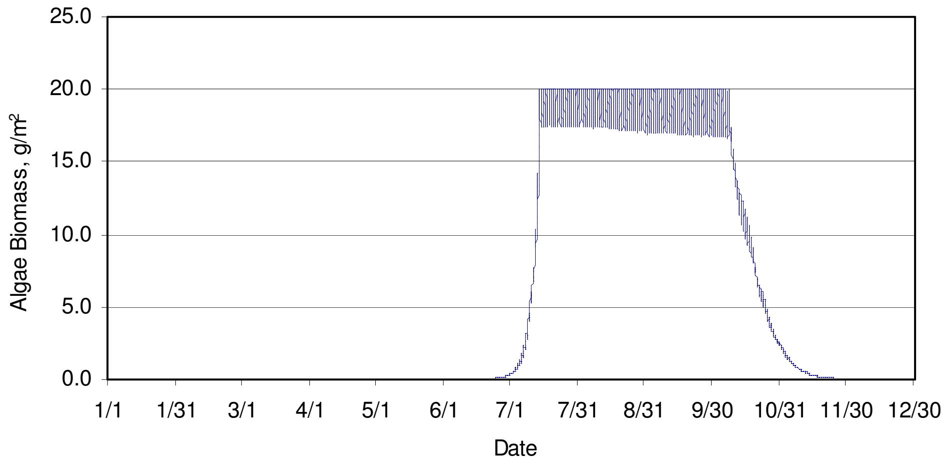


Figure 48. Algal biomass with global shade factor equal to 0.9

Altering Minimum Algal Biomass

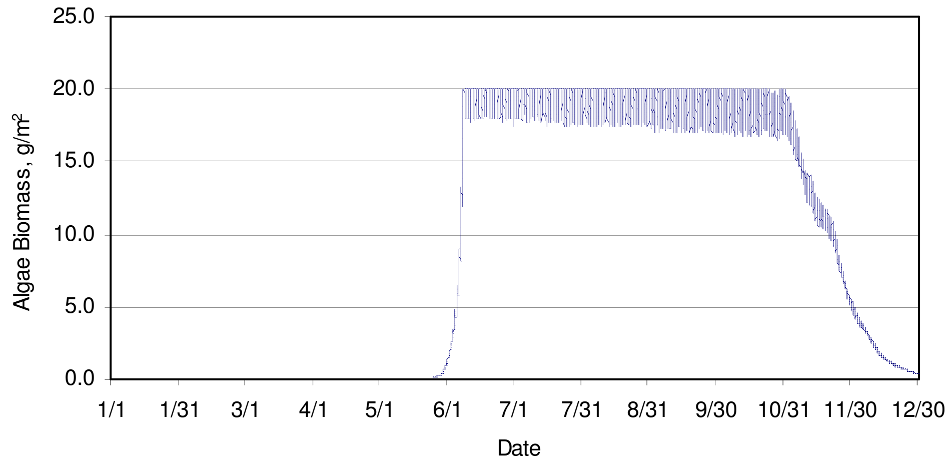


Figure 49. Algal biomass with minimum algal biomass equal to 0.0 g/m^2

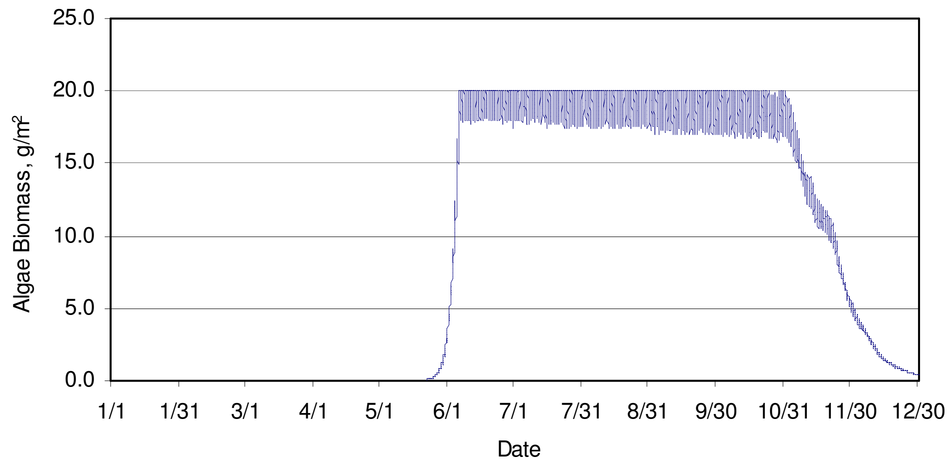


Figure 50. Algal biomass with minimum algal biomass equal to 0.2 g/m^2

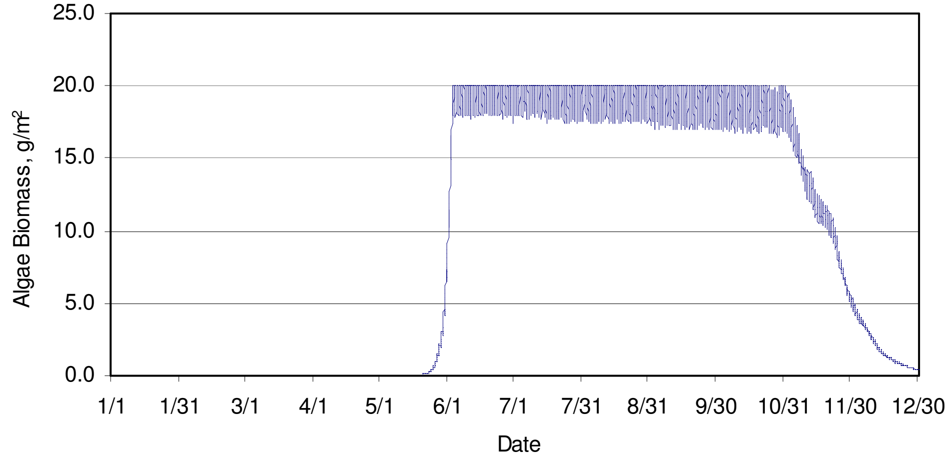


Figure 51. Algal biomass with minimum algal biomass equal to 0.5 g/m^2

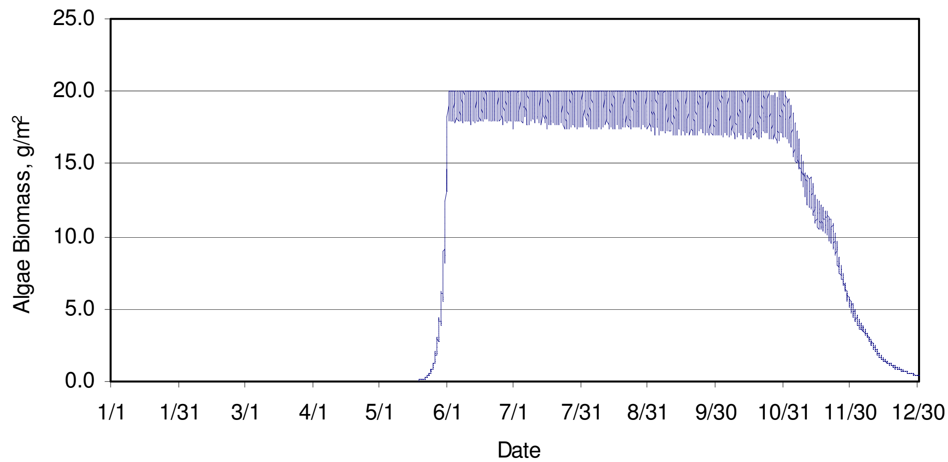


Figure 52. Algal biomass with minimum algal biomass equal to 1.0 g/m^2

Altering Maximum Algal Biomass

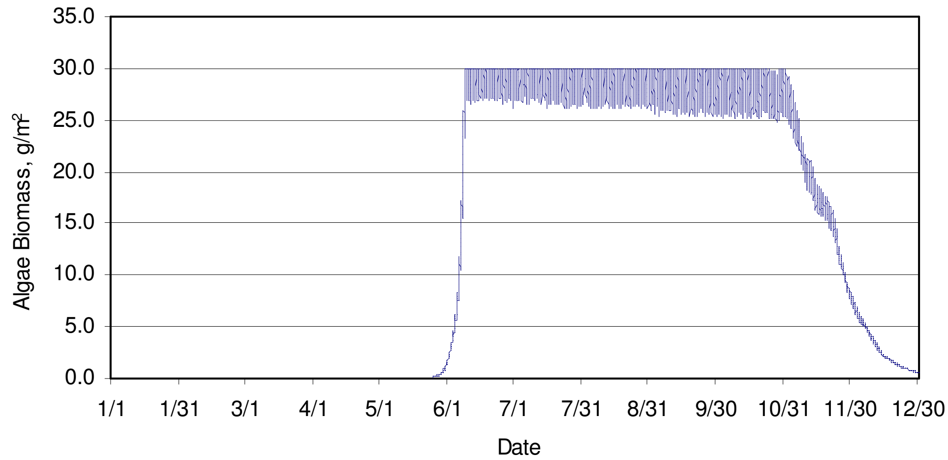


Figure 53. Algal biomass with maximum algal biomass equal to 30.0 g/m^2

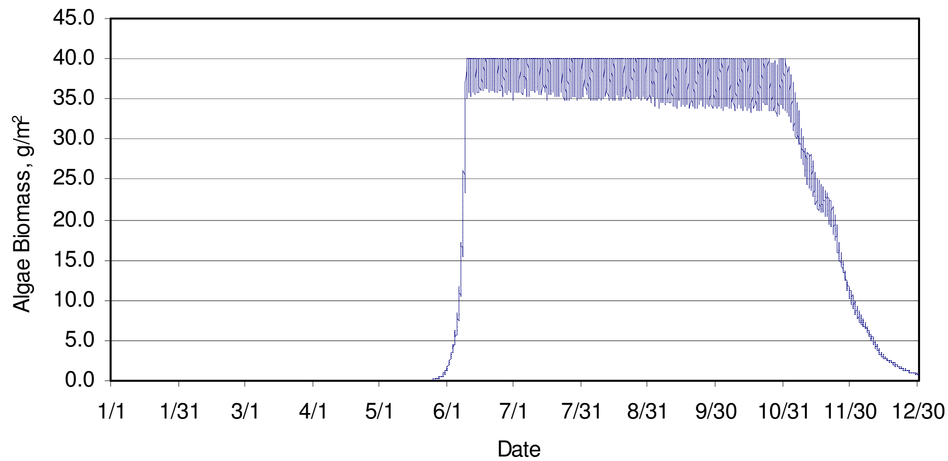


Figure 54. Algal biomass with maximum algal biomass equal to 40.0 g/m^2

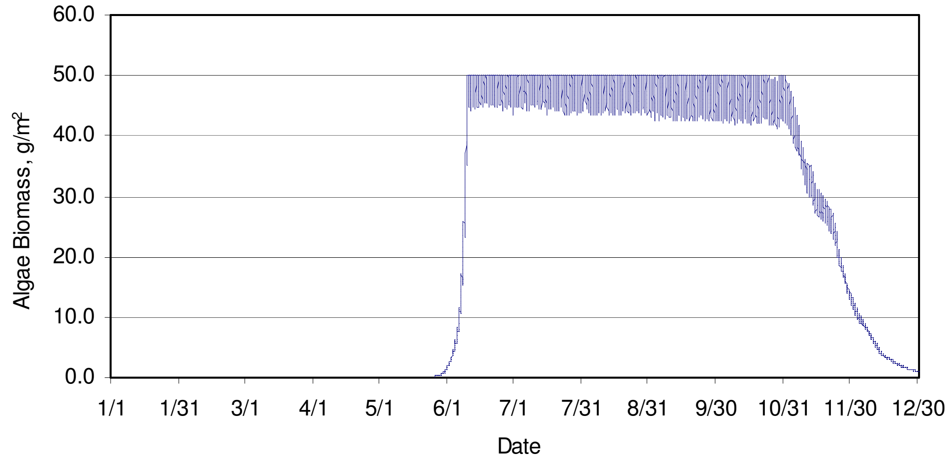


Figure 55. Algal biomass with maximum algal biomass equal to 50.0 g/m^2

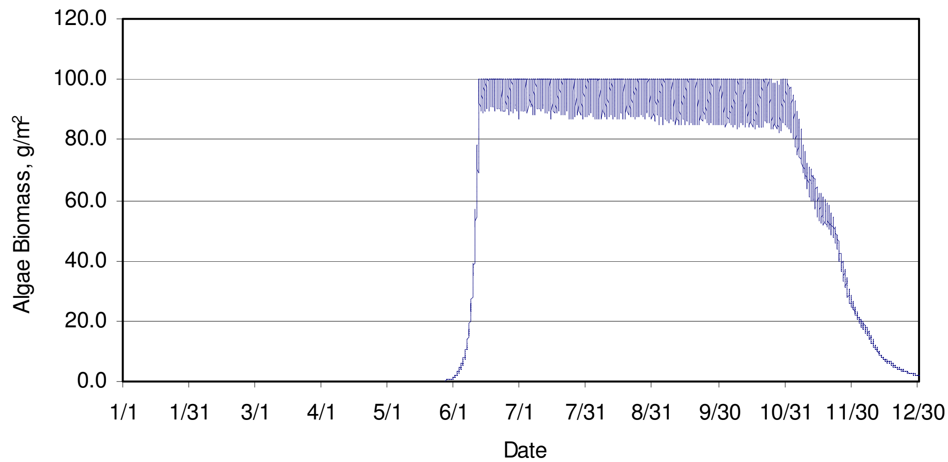


Figure 56. Algal biomass with maximum algal biomass equal to 100.0 g/m^2

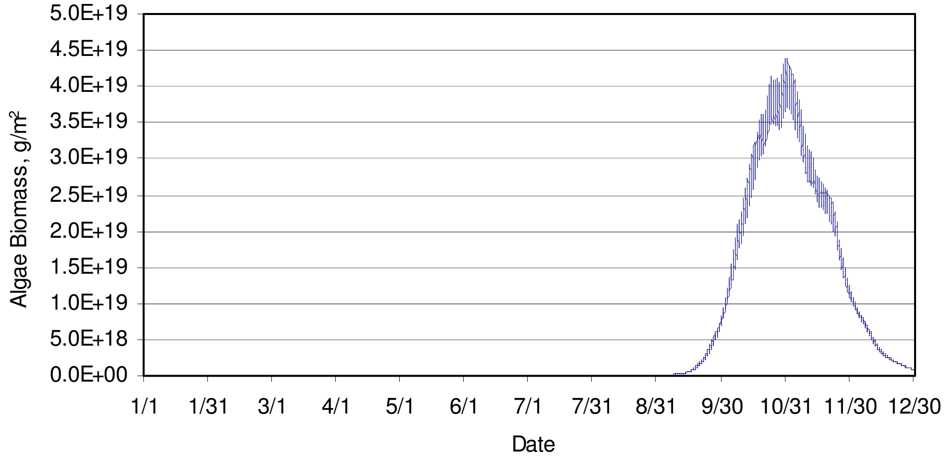


Figure 57. Algal biomass with maximum algal biomass equal to 1 x 10^{27} g/m^2

Altering Initial Algal Biomass

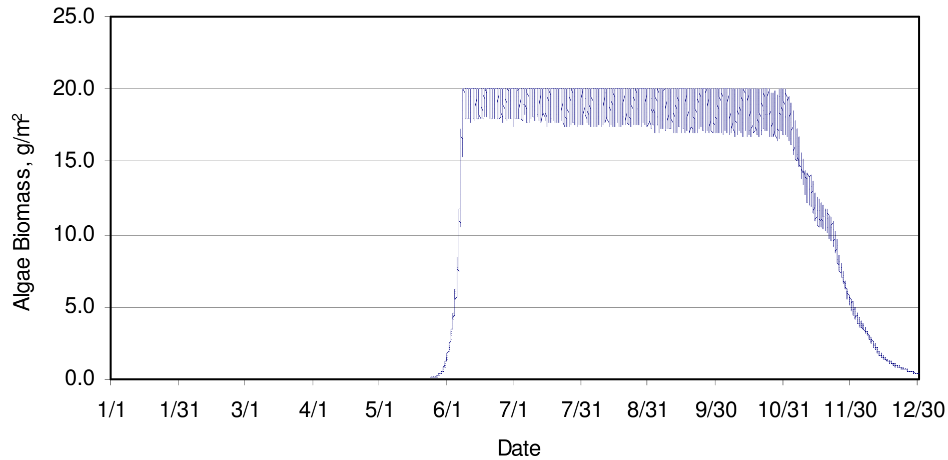


Figure 58. Algal biomass with initial algal biomass equal to 0.000 g/m^2

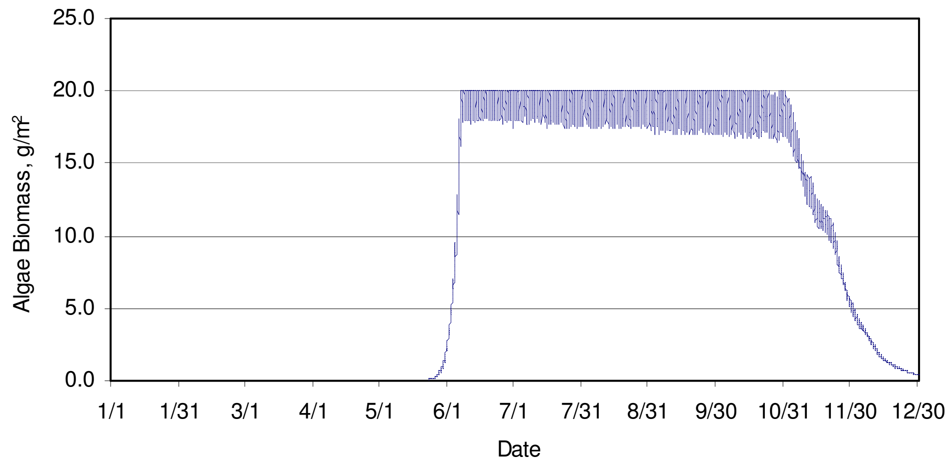


Figure 59. Algal biomass with initial algal biomass equal to 0.002 g/m^2

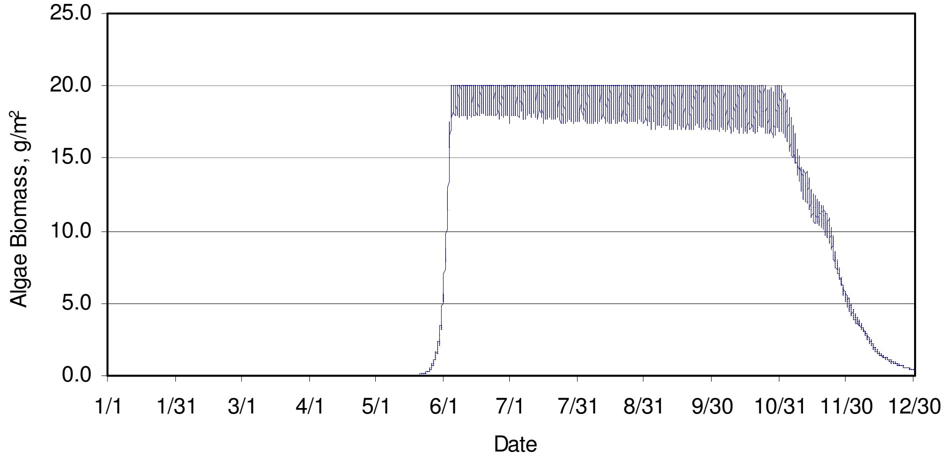


Figure 60. Algal biomass with initial algal biomass equal to 0.005 g/m^2

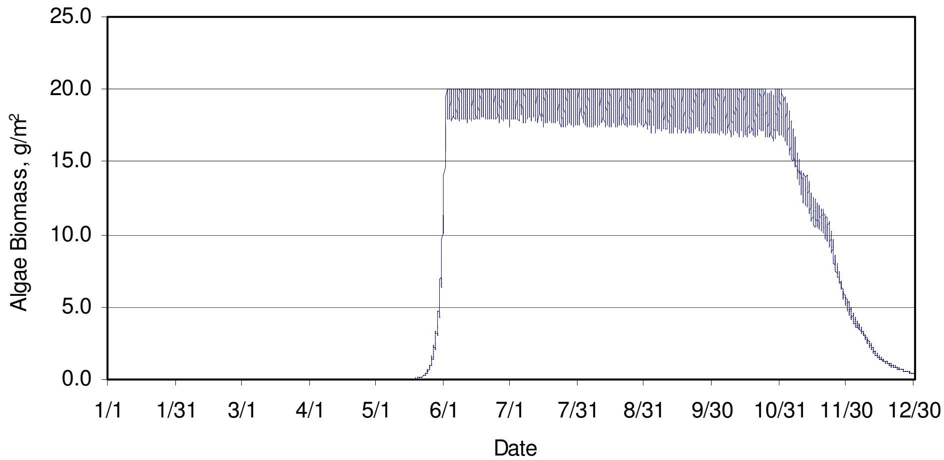


Figure 61. Algal biomass with initial algal biomass equal to 0.010 g/m^2

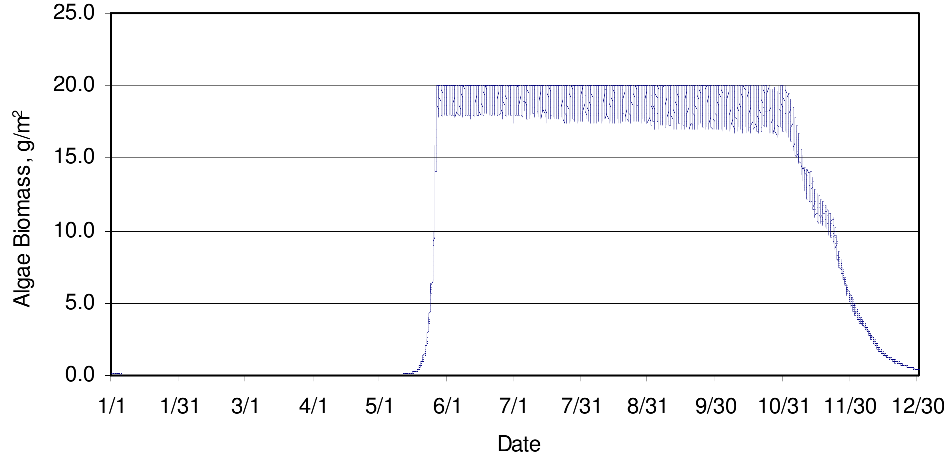


Figure 62. Algal biomass with initial algal biomass equal to 0.100 g/m^2

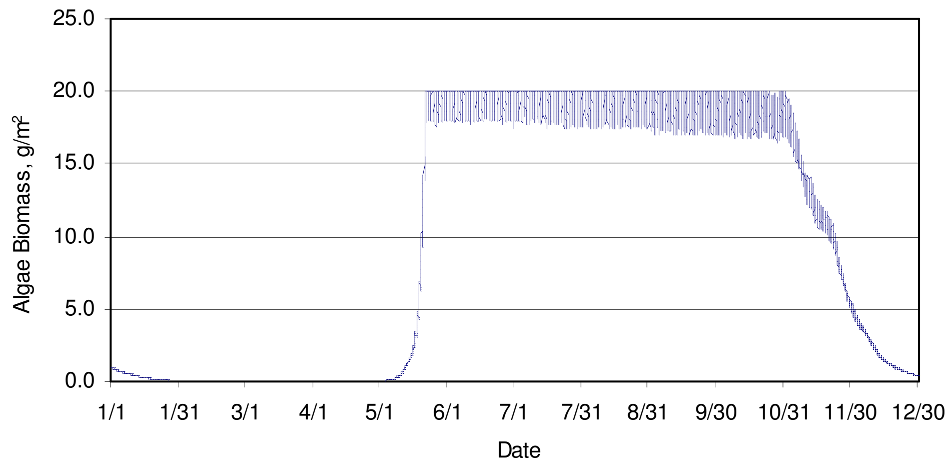


Figure 63. Algal biomass with initial algal biomass equal to 1.000 g/m^2